MIXING ZONES

BY

MARTIN P. WANIELISTA, PH.D., P.E.
ROBERT SMITH, P.E.
AL CASTRO, E.I.
AND
DAVID COOPER, PH.D., P.E.

UNIVERSITY OF CENTRAL FLORIDA
COLLEGE OF ENGINEERING
CIVIL ENGINEERING AND ENVIRONMENTAL SCIENCES DEPARTMENT
ORLANDO FLORIDA 32816

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PROJECT OFFICER
CHUCK ALLER, FDOT
LARRY BARFIELD, FDOT
HAYDEN BURNS BUILDING
605 SUWANNEE STREET
TALLAHASSEE, FLORIDA 32304

ABSTRACT

Mixing zones in streams and lakes were examined using appropriate mathematical models. Preliminary conclusions were developed on the extent of mixing zones resulting from stormwater discharges.

Two existing stream dispersion computer models were applied to a hypothetical stream to analyze mixing zone requirements. Mixing zone curves were developed to define acceptable and unacceptable regions of stream response for steady-state and transient conditions. The impact on these regions due to varying quantitative and qualitative characteristics of the stream and waste discharges were investigated. A sensitivity analysis was conducted on the response of a transient one-dimensional stream dispersion model.

A new finite element computational procedure was developed for dissolved oxygen depletion in lakes caused by stormwater runoff. The model can be modified to include metals. Specific reaction rate kinetics would have to be determined for this application.

Results of the stream and lake model simulations indicate that in many cases dissolved oxygen depletion due to stormwater is not a major problem in terms of immediate impact. These dissolved oxygen mixing zone depletions, however, do not consider long-term effects. Thus the research was limited to short-term effects.

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CHAPTER I

INTRODUCTION

The detrimental effects of point and non-point discharges into receiving waters has initiated considerable interest in the environmental engineering field. Since the waste assimilation capacity of these receiving waters involves complex chemical, physical, biological and hydraulic factors, the modeling of these factors to determine the assimilation capacity is a difficult undertaking.

Attempts to satisfy private and municipal industries, as well as the environmentally conscious segment of the populations, have resulted in the development of mixing zone strategies. An extensive literature review of surface water mass transport models has uncovered a lack of research in the area of mixing zone analysis, particularly in a quantitative approach.

The purpose of this work is to demonstrate how computer-based water quality models can be utilized to determine mixing zone requirements, and compliance to existing mixing zone standards, for stormwater discharges into a river, stream or lake. The result is the development of mixing zone curves or graphs for steady-state and transient modeling of the assimilative capacity of a stream and lake. Thus, future users can apply the results for stormwater management practices.

MIXING ZONE CRITERIA

It is believed that the State of Ohio was the first to develop a mixing zone criteria. The criteria applied to both Lake Erie and rivers, however other lakes in the state had no criteria. The State of Florida criteria are similar to those of Ohio but also are applied to lakes. As defined from the Florida Department of Environmental Regulation's (DER) point of view the reason for mixing zone is to allow the water quality adjacent to a point of discharge to be degraded so as to provide opportunity for mixing and thus, reduce the costs of treatment (DER, 1982). According to the rules of the DER Chapter 17-4 Article 17-4.244, a zone of mixing shall be determined based on consideration of the following:

1. The condition of the receiving body of water including present and future flow conditions and present and future sources of pollutants.

2. The nature, volume and frequency of the proposed discharge of waste including any possible synergistic effect with other pollutants or substances which may be present in the receiving body of water.

3. The cumulative effect of the proposed mixing zone and other mixing zones in the vicinity.

Mixing zones standards have been defined as follows:

1. A mixing zone shall not include an existing water supply intake nor include any other existing water supply intake if such mixing zone would significantly impair the purpose for which the supply is utilized.

2. A mixing zone shall not include a nursery area of indigenous aquatic life nor include any area approved by the Department of Natural Resources for shellfish harvesting.

3. In canals, rivers, streams and other similar water bodies, the length of a zone of mixing shall be 800 meters unless a shorter length is necessary to prevent significant impairment of a designated use. In no case shall a mixing zone be larger than that necessary to meet water quality standards.

4. In lakes, estuaries, bays, lagoons, bayous and sounds, the area of a mixing zone shall be 125,600 square meters unless a lesser area is necessary to prevent significant impairment of a designated use. In no case shall a mixing zone be larger than that necessary to meet water quality standards.

5. In a given water body, the cumulative mixing zone should not exceed the following limitations (see Table 1).

    a. In rivers, canals and other similar water bodies, 10% of the total length.

    b. In lakes, estuaries, bays, lagoons, bayous and sounds, 10% of the total area.

6. Additional standards on length and area which deline mixing zones in Class I, II and III waters are described in Table 1.

7. Selected water quality standards within mixing zones are shown in Table 2.

TABLE I

STORMWATER MIXING ZONE MATRIX

Water	Class (Mixing Criteria)	Length: Mixing Length (L_{mz})	Length: Cumulative Mixing Length (L_{cmz})	Area: Mixing Area (A_{mz})	Area: Cumulative Mixing Area (A_{cmz})	Additional Standards at Mixing Zone Area: Dissolved Oxygen (DO_{mz}) (ppm)	Additional Standards at Mixing Zone Area: Turbidity (T_{mz})
Lakes	I			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	II			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	III			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	IV			125,600 m^2	10% A_T		
	V			125,600 m^2	10% A_T		
Estuaries	I			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	II			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	III			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	IV			125,600 m^2	10% A_T		
	V			125,600 m^2	10% A_T		
Bays	I			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	II			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	III			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	IV			125,600 m^2	10% A_T		
	V			125,600 m^2	10% A_T		
Lagoons	I			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	II			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	III			125,600 m^2	10% A_T	4.0 $\geq$	75 JU $\leq$
	IV			125,600 m^2	10% A_T		
	V			125,600 m^2	10% A_T		
Canals	I	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	II	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	III	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	IV	800 m	10% L_T				
	V	800 m	10% L_T				
Rivers	I	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	II	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	III	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	IV	800 m	10% L_T				
	V	800 m	10% L_T				
Streams	I	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	II	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	III	800 m	10% L_T			4.0 $\geq$	75 JU $\leq$
	IV	800 m	10% L_T				
	V	800 m	10% L_T				

Class I: Public Water Supplies
Class II: Shellfish Harvesting
Class III: Recreation
Class IV: Agricultural and industrial water supply
Class V: Navigation, utility and industrial use

TABLE 2

WATER QUALITY STANDARDS (mg/l) IN MIXING ZONES

	I-A Potable Surface Water Supplies	II Shellfish	III Recreation
Cadmium	+.0008/ .0012	.005	+.0008/ .0012
Copper	.03	.015	.015/.030++
Iron	.3	.3	1.0
Lead	.03	.05	.03
Silver	.07	.05	.07/.05++
Zinc	.07	1.0	.03
D.O. 24 hr. avg.	5*	5**	5**

+If hardness ≤/50 mg/l $CaCO_3$/≥150 mg/l $CaCO_3$

*and never be less than 1.5 milligrams/liter

**and never be less than 4 mg/l

++Freshwater/marine water

STREAMS AND RIVERS

The steady-state program used for streams is the two-dimensional dispersion model (TWOD) for point source discharges into rivers. It was developed by the Florida Department of Environmental Regulation. To allow wastewater treatment plant operators to meet existing dissolved oxygen (DO) and mixing zone length standards and cross-sectional area recommendations at varying discharge capacities, mixing zone curves were derived for a hypothetical stream with constant hydraulic, kinetic and water quality parameters. The curves were compared to each other to observe the effect of varying initial effluent and receiving water BOD and DO concentrations on the mixing zone length standard, therefore, defining acceptable and unacceptable regions of compliance for each combination.

In addition, mixing zone surface and cross-sectional area requirements as a function of receiving water flowrate were determined for constant stream and wastestream qualitative characteristics. Again, acceptable and unacceptable regions of compliance were defined for cross-sectional area requirements.

The transient non-point computer program utilized for streams is the one-dimensional dispersion model, SWOPS, developed by the U.S. Environmental Protection Agency. The program was modified to be more useful for frequent applications in BOD and DO dispersion, as well as mixing zone analysis. Equations were formulated and introduced in the program to calculate the predicted time and distance downstream where a particular concentration of BOD and DO occurs. The equations developed consistent with the Lagrange coordinate system initially employed in the model.

An example is presented to illustrate the transient mixing zone length requirements for a specific stormwater event entering a hypothetical stream using SWOPS. Mixing zone length and average DO concentration requirements were plotted versus time after discharge to determine the magnitude and duration of violations to existing mixing zone length and average DO standards. Similar approaches can be conducted in field studies to determine proper mixing zone and treatment control strategies.

LAKES

A computational procedure for numerical simulation of the depletion of dissolved oxygen resources in small lakes caused by stormwater runoff had been devised and programmed for the digital computer. The model, called LADE, computes dispersion and decay of a biodegradable pollutant such as BOD or COD. Also included in the simulation is the utilization of dissolved oxygen by diffusion over the lake surface from atmospheric air. The model is quasi three-dimensional and time dependent.

The reaction terms in the model can be modified to include other water quality parameters. Interest in metals accumulation to include reaction kinetics is evident. The model is general in nature and other water quality parameters and kinetics can be incorporated.

The results of simulating BOD_5 and dissolved oxygen using the LAKE model were compared with an analytical solution. Results were similar. Use of the model indicates no short-term dissolved oxygen effects from stormwater discharges.

CHAPTER II

LITERATURE REVIEW

GENERAL OVERVIEW OF WATER QUALITY MODELING

Historically, water quality modeling of surface waters (i.e., streams/ rivers, lakes and estuaries) has branched into two distinct methodologies, each being defined by the approach undertaken, in simulating water quality response to a contaminated discharge on the system. The first, or deterministic modeling, was initially the work of H.W Streeter and H.B. Phelps (1925) on the Ohio River.

Deterministic modeling assumes that a surface water quality response can be described by a distance and/or time averaged parameters(s) at a particular point downstream from the point of pollution discharged. Its ease of applicability and comprehension has made this approach populate to researchers and legislators in assessing violations to existing water quality standards. Cembrowicz, Hann, Plate and Schultz's (1978) literature survey concluded that the trend is still strong in employing lump sum steady-state, deterministic formulations that summarize the effect of the various self-purification processes.

Due to the pioneering work of Streeter and Phelps (1925), the classical BOD/DO relationships have been widely accepted in deterministic modeling as the most important water quality response parameters. This has led many researchers to criticize the classical BOD/DO relationship since no provisions were made to account for the discrepancy between laboratory BOD determination and natural processes, including the effects of turbulence, sedimentation, adsorption and toxic substances upon the interdependencies and interactions with other quality parameters (Cembrowict et al. 1978).

As a result, deterministic modeling has been expanded in recent years to include detailed mathematical formulations with an increasing number of system parameters describing the physical, chemical and biological mechanisms involved in natural purification. In addition, transient models utilizing the deterministic approach have been derived to account for the temporal variations of a pollutant as it spreads in a surface water.

As an example, Dobbins (1964) outlined the various mechanisms affecting the BOD and DO relationships in a stream as follows:

1. The removal of BOD by sedimentation or adsorption.

2. The addition of BOD along the stretch by the scour of bottom deposits or by diffusion of partly decomposed organic materials from the benthal layer into the water above.

3. The addition of BOD along the stretch by local runoff.

4. The removal of oxygen from the water by diffusion into the benthal layer, to satisfy the oxygen demand in the aerobic zone of this layer.

5. The removal of oxygen from the water by purging action of gases rising from the benthal layer.

6. The addition of oxygen by the photosynthetic action of plankton and fixed plants.

7. The removal of oxygen by respiration of plankton and fixed plants.

8. The continuous redistribution of both the BOD and oxygen by the effect of longitudinal dispersion.

9. BOD and DO removal by bacterial oxidation of organic matter.

10. The replenishment of DO by reaeration at the surface.

Since pollution is a stochastic process, many researchers feel that surface water quality standards based on deterministic approaches are not adequate in preventing the quality of an aquatic environment from deteriorating to levels harmful to aquatic life. Many situations have occurred where the mean DO level in a water body has satisfied the standard, but natural variations have caused the DO concentrations to fall fare below the standard for prolonged periods of time, thus causing fish kills.

This has led researchers to develop stochastic models to simulate the temporal and spatial distribution of DO and pollutants as they would occur in nature. A good stochastic model can predict not only the average concentration of a pollutant and the associated DO but also the variability of pollutant or the DO concentration about its average value. In addition, it should be able to predict accurately the proportion of the time that DO will be below any given concentration level (EPA 1971a). Therefore, it is the hope of these researchers that appropriate standards can therefore be implemented based on this criteria.

DETERMINISTIC MODELS

Mahoumoud and Ahmad (1979) applied the Streeter-Phelps Model to the River Tigris in Iraq in an attempt to assess the causes of poor water quality observed in the river. DO concentrations for the Iowa River were predicted with this model to estimate the effects of agricultural land runoff and wastewater discharges on water quality (Wallace and Dague 1973). The study was conducted to simulate low river flow conditions to evaluate the worst possible river condition. They concluded that the only cause of low DO levels in the Iowa River seems to be land runoff.

Since the Streeter-Phelps equation considers deoxygenation and reaeration to be the only mechanisms effecting the DO balance in a surface water system, skeptics have modified or discarded this model on grounds that a DO balance involves more complex natural mechanisms in addition to the two mentioned.

Dobbins (1964) included the effects of dispersion, BOD removal by sedimentation or adsorption. The removal of DO by benthal demand and the effects of aquatic plants on the Streeter-Phelps sag equation are also considered. His model will be presented in greater detail in a later section. Comparison of the sag-curves calculated by the Streeter-Phelps and Dobbins equations have consistently shown that the Dobbins Model is more efficient in estimating the minimum DO level and DO recovery in river systems (Mahmoud and Ahmad 1979).

The effects of nitrogenous BOD (NBOD) on the DO concentration in streams and rivers have been included in modified Streeter-Phelps models. Bathala, Das and Jones (1979) assumed the nitrogenous BOD also follows first-order kinetics.

Many researchers have favored using the classical steady-state one-dimensional mass balance equation for an ideal plug-flow reactor in modeling the assimilative capacities of streams and rivers for point discharges. Like the classical Streeter-Phelps Model, the ideal plug-flow model assumes that bulk flow proceeds throughout the system in an orderly uniform manner. There is no mixing due to concentration gradients in the longitudinal direction. The contents vary along the axis of flow due to advective and reactive forces. The unsteady-state equation can be represented as follows (Weber 1972):

$$\frac{\partial C}{\partial t} = -v_x \frac{\partial C}{\partial x} + r(C) \qquad (1)$$

and the steady-state equation as:

$$0 = -v_x \, dC/dx + r(\overline{C}) \qquad (2)$$

where:

C = mass or concentration of the constituent under investigation

v_x = velocity component along the longitudinal direction (X)

$r(\overline{C})$ = reactive term representing the decay of C_i

Meadows, Weeter and Green (1978) developed a model to determine the water quality impact under steady-state conditions from non-point sources in small streams. The model computes the spatial variations of BOD and DO for gradually varying stream flow conditions. The model differs from the Streeter-Phelps relationships due to the inclusion of a separate term, representing the addition of BOD and DO from stormwater in the BOD and DO mass balances. The uptake of DO by plant respiration is also addressed in the DO equation. Partial differential equations are incorporated into the governing mass balance equations to drive the steady-state solutions for BOD and DO profiles downstream.

The model was used to assess stormwater water quality impact on Bruch Creek, Washington County, Tennessee. Preliminary results revealed that nutrient control might be essential to minimize the effect on low DO levels due to aquatic plant respiration at night. Also, stormwater contributions to water quality in small streams can be so adverse that studies should be designed to isolate, monitor and quantify these sources.

Since the solution to the partial differential equation is difficult to obtain mathematically, most researchers have opted to model in the steady-state region. Historically, few have attempted to model streams and rivers as one-dimensional unsteady-state (transient) plug-flow reactors.

Thomann (1973) successfully modeled a stream using a transient one-dimensional plug-flow model with time variant input. He did this by representing the dynamic response in terms of the frequency response of single and coupled (BOD-DO) water quality variables with first-order reaction kinetics.

Thomann (1973) also applied the same methodology to a transient one-dimensional dispersion model and compared the performances of both models to single and coupled variable systems for deciding whether a no-mixing plug-flow model or a dispersion model is suitable when the problem context involves a time varying waste input. The results indicated that, when waste load inputs vary with periods of 7 days or less, the effects of small amounts of dispersion on the amplitude of the water quality response can be significant. The effects of dispersion cannot generally be neglected for large, deep rivers.

DISPERSION-ADVECTIVE MODELS

As with plug-flow advection models, dispersion models for aquatic systems are derived from conservation of mass principles. Dispersive models differ from plug flow models by the addition of a dispersion term to the advective term, representing the total flux of mass into or out of a system. The flux due to dispersion is assumed to be proportional to the concentration gradient in the direction of decreasing concentration (EPA 1971b). This "Fickian" relationship defines the transfer of mass from regions of high concentrations to one of low concentrations. Factors such as turbulent diffusion, velocity gradients, tidal effects and density differences, when present, contribute to the total spread of mass.

The complete material balance for the transport and dispersion of a non-conservative substance in an aquatic environment is given by the following equation (DER 1979):

$$\frac{\partial C}{\partial t} + \mu \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} + W \frac{\partial C}{\partial Z} = \frac{\partial}{\partial X} [E_X \frac{\partial C}{\partial X}] + \frac{\partial}{\partial Y} [E_Y \frac{\partial C}{\partial Y}] + \frac{\partial}{\partial Z} [E_Z \frac{\partial C}{\partial Z}] - KC + S \quad (3)$$

where:

X, Y, Z = the components of the position vector (X-longitudinal, Y-latitudinal and Z-vertical)

μ, V, W = components of the velocity vector in the X, Y and Z direction, respectively

C = concentration of the constituent

E_X, E_Y, E_Z = eddy diffusion or turbulent diffusion coefficient in the X, Y and Z direction, respectively

K = reaction rate constant

S = any distributed sources and/or sinks of the constituents

Equation 3 can be used to represent the material balance for a consecutive substance by simply neglecting the last two terms on the right-hand side (McQuivey and Keeter 1976).

Due to the mathematical complexities encountered in solving Equation 3, many researchers have incorporated simplifying assumptions or have developed numerical techniques to model the dispersion of pollutants in rivers and streams.

TRANSIENT MODELS

Thomann (1963) developed a mathematical model for describing the time variations of DO in a finite number of sections in an estuary. A system analysis technique was incorporated to develop linear response equations for DO resulting from the imposition of general input time-variable forcing functions (i.e., sewage input, dispersion, advection, reaeration, deoxygenation and respiratory and photosynthetic action of aquatic plants). The equations can then be expressed in matrix notation and easily solved with a computer.

Dresnack and Dobbins (1968) applied a finite-difference technique to develop the numerical solution of the modified forms of Equation 3. Explicit and implicit methods were utilized to develop the numerical scheme. Each term in the governing equation was substituted with a comparable finite difference representation. This allows the equation to be implemented at every point in a network (or mesh) simulating the stream reach under consideration. The network then allows a computer to numerically solve the equation at every point, therefore, allowing the BOD and DO profiles to be generated in a stream receiving diurnally and spatially varying inputs.

This model was successfully applied in the Waikato River in New Zealand. Concern about its deteriorating water quality due to partially treated commercial and domestic effluent, forest and pasture drainage (Rutherford 1977) was voiced. The study goals were to model the diurnal DO levels and to predict the likely effects of damage to the inhabiting aquatic plant communities in this river. The model was calibrated solely by comparing observed and predicted DO levels. Phytoplankton, Periphyton and Macrophyte plant communities were analyzed by developing equations that defined the oxygen production and consumption resulting from each community. The results were included in the Dresneck-Dobbins DO equation. It was found that phytoplankton are the principal contributors to the observed diurnal variations in DO levels. The model was unable to predict the effects of pollutants on plant communities in its present form.

To study the behavior of a dispersing tracer cloud in a river, Ward (1973) effectively reduced the conservative form of Equation 3 to a transient two-dimensional diffusion equation by depth-averaging. The resulting equation considered the longitudinal as well as the transverse dispersion of a dye in a river reach. His assumption is valid since most rivers are much wider than they are deep (Cleary and Adrian 1973 and Ward 1973). The goal was to determine the length between point of injection and the point where the tracer cloud has had time to spread across the river. This length herein referred to as the mixing length determines the distance required to achieve complete mixing across the channels.

By assuming that the flow is uniform and steady, and averaging the concentrations along the X-axis over a distance, L, greater than the length of the tracer cloud, the two-dimensional equation was again reduced to give a transient one-dimensional diffusion equation assuming that the river depth and mixing are constant. The equation is representative of situations encountered in a wide, shallow canal rather than a river.

Two- and three-dimensional transient modeling of water pollution, though more accurate than the basic one-dimensional approach, introduces additional mathematical and analytical complexities. The analytical problems include the determination of multi-dimensional eddy turbulent diffusion coefficients as well as the complicating no-flux boundaries necessary in modeling surface water bodies. To simplify matters, mathematical procedures such as the method of moments, statistical variance analogies, gradient search, and least-square fit methods have been used to determine the required diffusion coefficients from observed data. The assumption of infinite longitudinal, vertical and lateral dimensions have been utilized to neglect no-flux boundary conditions, therefore, simplifying the procedure in determining the analytical solutions (Henry and Foree 1979 and Shen 1978).

As a result of their work, Clearly and Adrian (1973) presented a solution technique and analytical solutions to the two- and three-dimensional, unsteady-state, convective-diffusive partial differential equations which describe the concentration distribution of a tracer dye released as an instantaneous source (line or point).

The solutions were obtained by integral transform methods and are in the form of infinite series which converge rapidly and may be easily programmed. The governing two- and three-dimensional equations were coupled with equations expressing the boundary conditions at each surface of interest to arrive at the solutions. The finite and infinite dimensions were transformed out of the equation by a finite Fourier transform and a complex Fourier transform, respectively as required. The two- and three-dimensional models were applied to a hypothetical river to obtain concentration distributions of dye for centered and off-centered sources to test for model effectiveness.

Henry and Foree (1979) incorporated a two-dimensional transient disperson model developed to by-pass the difficulties associated with Clearly and Adrian's model (1973). The model describes dispersion from a surface source in which all error functions and integrals are approximated, therefore, allowing temporal and spatial concentration profiles to be obtained directly by inserting the appropriate parameter values. Iterative techniques are not required since they have already been incorporated into the development of the final equation. Dispersion is again considered constant along the longitudinal and lateral directions, and lateral mass transfers only by dispersion, therefore, providing the one-dimensional velocity assumption as before. Three applications of the model were illustrated and results showed that the model can be efficiently used for environmental impact studies.

To describe the concentration distribution of a substance or heat in a time-dependent flow field, Yeh and Tsdi (1976) analytically solved a transient, three-dimensional turbulent diffusion equation. The governing equation, similar to Equation 3 is written in terms of excessive temperature or substance concentration above that of the ambient fluid. First-order, decomposition and generation rates are assumed. The averaged one-dimensional velocity can be a function of time, thus allowing the modeling of tidal-influenced water bodies. Initial and boundary conditions for the lateral and vertical dimensions are also specified.

The analytical solutions are derived using Green's theorem. Model results were compared to field measurements taken for conservative discharges such as dye and non-reactive substances. The results agreed with the data, therefore, illustrating the model's capability in simulating the space and time variation of dye concentrations.

APPLICATIONS TO MIXING ZONES

Mixing Zone Standards

Concerned with the protection of aquatic biota against chemical or thermal discharges into a receiving water, the National Technical Advisory Committee to the Secretary of the Interior (1968), suggested mixing zone criteria for streams, lakes and estuaries. The mixing zone would be an area set aside from the remainder of the receiving water where mixing of chemical and thermal effluents would allow the zone's water quality to violate existing ambient standards. The mixing zone area(s) would be allocated such that a sufficient zone of passage would exist to allow aquatic biota to migrate without hazard. The water quality in the zone of passage would meet governmental standards.

Though leaving the shape, sizing and location of mixing areas to the discretion of proper administrative authority, the report recommended that for any stream or estuary the area outside the mixing zone "should contain preferably 75 percent of the cross-sectional area and/or volume of flow of the stream or estuary."

The Committee went on to recommend appropriate DO standards for the zone of passage. To summarize:

1. For a diversified warm-water biota, the daily DO concentration should be above 5 mg/l, assuming normal daily and seasonal variations above this concentration. The DO may vary between 5 mg/l and 4 mg/l for short periods of time if the water quality is favorable in all other respects.
2. For the cold water biota, DO concentrations should be at or near saturation. The recommended daily DO should be near 6.0 mg/l for short periods if water quality is favorable.

Many states have adopted these standards or have administered stricter regulations on mixing zones. For instance, Wisconsin restricts the surface area of mixing zones to 10% of the total surface area in lakes. Allowable temperature rise at the edge of a mixing zone for thermal loadings range from 5° F (2.78° C) for rivers and 3° F (1.87° C) for lakes and impoundments (Paily 1981).

Mixing Zone Modeling

A review of the literature has revealed that minimal research has been conducted on mixing zone requirements for point or non-point discharge into an aquatic environment. In fact, most of the past analysis has been conducted on heated discharges from point sources into rivers. There is an immense deficiency in modeling mixing zones with respect to the water quality impact on DO concentrations in rivers or lakes, especially impacts due to non-point contributions.

A two-dimensional steady-state model was developed by the State of Florida Department of Environmental Regulation to assess mixing zone requirements due to point source discharges into a river by considering resulting variations of DO levels downstream from the injection point (Florida DER 1979). This model will be discussed in detail in a later chapter.

Stefan and Gulliver (1978) derived theoretical relationships for the maximum width, maximum length and total surface area of a mixing zone resulting from the shoreline discharge of a heated effluent into a shallow and wide stream. The mixing zone was defined as a volume of water enclosed by an isotherm of specified strength. Since shallow rivers are vertically well mixed and transport of a substance from the shoreline to the center of the stream is usually slow, depending on initial discharge momentum and the flow rate of the river, it was necessary to incorporate two-dimensional analytical diffusion equations, derived from semi-empirical techniques.

Paily (1981) demonstrated how field or site specific data from thermal plume surveys can be applied to derive mixing zone models. Incorporating dimensional analysis techniques relationships were derived to determine plume center line distance, surface area and width as functions of river temperature, dilution ratio and other geometrical and hydraulic characteristics. As with the previous model, a case study was conducted to demonstrate the versatility of this approach. Summer and winter low flow conditions were simulated to develop temperature profiles due to heated water discharges from an existing steam-electric power plant operating at various stages of full loading, to determine if mixing zone standards are met or violated under these conditions. Therefore, the model can be used by a regulatory agency in assessing water quality conditions at all power plant outfalls.

A similar approach was used by Parr and Sayre (1981) to determine mixing zone cross-sectional area and discharge from a multi-part diffuser. Empirical equations relating mixing zone cross-sectional area and discharge of individual port diffusers were developed and combined to derive mixing zone requirements for multi-part diffusers as a whole. Again, the three basic geometrical characteristics of mixing zone areas mentioned previously were used as the basis for developing these equations. Empirical formulas for jet behavior in confined, flowing receiving waters were incorporated in the model and predictive equations and graphs for mixing zone area and diffuser discharge were presented. Comparisons between predicted and experimentally measured mixing zones were made giving good results.

CHAPTER III

TWO-DIMENSIONAL STEADY-STATE MIXING ZONE MODEL (TWOD)

INTRODUCTION

The steady-state Two-Dimensional Mixing Zone Model (TWOD) was developed primarily to examine the validity of the water quality based effluent limitations (WQBEL) listed on state permits. The WQBEL were defined before the concept of mixing zones was developed, and their use in defining effluent limitations for each permitted surface water discharge in mixing zone analysis were questioned. TWOD was compared to the River Model, which was then used extensively in defining WQBEL, to find if modifications to River (and therefore WQBEL) were required to consider mixing zones or if the river model results should be disregarded. It was concluded that the TWOD model should be used in conjunction with the river model for the consideration of water quality based effluent limits and mixing zones (Florida DER 1979).

MODEL DEVELOPMENT

The contents of this section represents a summary of the material contained in DER Model (1979). The first step in the development of TWOD is to consider the basic equation describing the transport and dispersion of a constituent in an aquatic environment. It would be extremely difficult to solve Equation 3 in three dimensions for unstead-state conditions. Therefore, Equation 3 was simplified by incorporating the following assumptions:

1. The system is in steady-state ($\partial C/\partial t = 0$).
2. The receiving body of water is vertically homogeneous so that the equation can be integrated vertically ($\partial C/\partial t = 0$).
3. Lateral and vertical velocities are negligible (i.e., $V = W = 0$).
4. The downstream velocity (μ) varies only across stream (i.e., $\mu \equiv \mu(y)$).
5. The water depth (Z) varies only across stream (i.e., $Z \equiv Z(y)$).
6. The eddy diffusion in the X-direction is negligible compared to the advection (i.e., $E^X = E^Z = 0$).

The boundary conditions given in Equations 6 and 7 were incorporated into Equation 4 to analytically arrive at an exact solution. The equations for the coupled BOD-TKN-DO system representing the impact of sewage discharges on instream dissolved oxygen concentrations are given in Table 3.

MODEL INPUT REQUIREMENTS

The purpose of this section is to describe and discuss the physical parameters needed as input data in TWOD. The researcher can simulate desired stream and surface discharge conditions. This can enable him to determine mixing zone control strategy by assessing the impact of discharging a waste-stream of known strength into an existing stream or natural waterway.

It must be remembered that TWOD is a steady-state model, therefore, it is independent of time considerations. The results given by TWOD represent conditions existing in a stream when the system reaches equilibrium after a long time period following steady discharge of pollutant. It does not take into account the transient nature of transport and dispersive forces which can be very important in predicting short term environmental impact on aquatic life. It is, therefore, important to realize that all input parameters are constant with respect to time.

The model assumes that the waste is introduced as a line source. This is reasonable in light that the effluent in the immediate vicinity of the point of discharge will maintain its integrity until its momentum is overcome by turbulent spreading. This implies that a pipe discharge perpendicular to the flow will spread across-stream for some distance prior to moving down-stream. If the flow of the stream is low compared to the discharge, lateral dispersion will predominate at distances near the injection point, over downstream mixing.

The stream section to be analyzed by the model must be assumed rectangular in surface area. The programmer can specify the width and length of the section by inputing the number of lateral and longitudinal distance increments desired, respectively. A maximum of twenty-one cross-sectional increments can be specified. The length of each increment is constant and specified by the programmer. Twenty-one cross-sectional depths and velocities can, therefore, be specified for each cross-sectional increment. Both of these parameters represent average values and can be variable or constant depending on the characteristics of the stream being simulated. There is no limit to the number of longitudinal increments that can be specified.

TABLE 3

MASS BALANCE EQUATIONS FOR DISSOLVED OXYGEN,
BIOCHEMICAL OXYGEN DEMAND AND TOTAL KJELDAHL NITROGEN

Equations:	
DO(D)	$\frac{U\partial}{\partial x}(D) = \frac{\partial}{\partial y}[E_y \frac{\partial (D)}{\partial y}] - K_A(D-D_s) + F_L K_D B + F_N K_N T$
BOD_5(B)	$\frac{U\partial}{\partial x}(B) = \frac{\partial}{\partial y}[E_y \frac{\partial}{\partial y}(B)] - K_R B$
TKN(T)	$\frac{U\partial}{\partial x}(T) = \frac{\partial}{\partial y}[E_y \frac{\partial}{\partial y}(T)] - K_N T$

where:

$F_L = 1/(1-\exp(-5K_R))$

$F_N = 4.57$

C_S = DO at 100% saturation

K_A = reaeration rate

K_R = BOD oxidation rate

K_D = BOD oxidation and settling rate

K_N = TKN oxidation rate

SOURCE: Florida Department of Environmental Regulation. Methods for the Consideration of Water Quality-Based Effluent Limitations and Mixing Zones, June 1979.

The initial stream conditions are introduced next. The existing quality of the receiving water is given by inputing the initial flow, dissolved oxygen, BOD and TKN concentrations, in addition to stream temperature and the saturation constant for DO at that temperature. These physical parameters should be similar to real conditions existing in the stream being simulated. Kinetic parameters representing reaeration rate, BOD oxidation rate, BOD oxidation and settling rate, and TKN oxidation rate constants are specified next.

The flow rate, DO, BOD and TKN concentrations must be included. Finally, the length of the line source (pipe) is specified by the lateral coordinates of the inlet and outlet of the source.

Sensitivity tests conducted at the Florida State Department of Environmental Regulation (DER) illustrated that TWOD is sensitive to the size and location of the surface diffuser (pipe). In general, long diffusers are preferred over short ones to enhance mixing capabilities. Long diffusers allow the plume to disperse over a wider area in the stream and decrease the magnitude of concentrating the constituent along the nearest bank of the stream, especially in low stream flow rate conditions. Also, stream centered diffusers are preferred over bank diffusers since centered outfalls have about twice the area in which to disperse (Florida DER 1979).

TWOD is also sensitive to changes in the lateral disperson coefficient, E. As expected, as the lateral dispersion for a given effluent discharge decreases for both boundary and mid-stream diffusers. The model sensitivity to changes in E are less pronounced in the former case when compared to the latter one. Curve fitting of mixing zone length to E shows a linear relation for boundary diffusers and a non-linear relation for mid-stream ones. The results are expected since an increase in E corresponds to an increase in the area (laterally) available for mixing purposes (Florida DER 1979).

MODEL OUTPUT REQUIREMENTS

Once the necessary parameters described in the preceding section are introduced to TWOD, the program proceeds to solve the exact solutions to the mass balance equations shown in Table 3. The procedure is a step series solving the BOD and TKN equations first and finally the DO equation for each coordinate (longitudinal plus lateral increments) in the solution reach.

The physical output provided by TWOD begins with the listing of the input parameters described previously. Next, it illustrates the cross-sectional geometry of the stream in question for all twenty-one cross-sectional segments prescribed. The lateral position (distance), average depth, average velocity, calculated flow rate and lateral dispersion coefficient for each segment are given.

The model results showing the DO, BOD and TKN concentration for each point in the rectangular stream reach follow. The results are given in tabular form as a matrix corresponding to the number of longitudinal and latitudinal distance increments specified. Each increment is represented as a real distance, therefore, each point in the solution reach corresponds to a position downstream and across-stream in the solution reach. In addition, the average lateral concentration for each constituent is shown for each downstream increment. The dissolved oxygen results also give the area and length of mixing zone required in addition to the minimum and total average DO concentration experienced in the stream segment analyzed.

If desired, a contour plot of DO, BOD and TKN concentration ranges in the stream segment can be provided. A program control variable depending upon the number (1 or 0) assigned to it instructs TWOD to provide a contour plot or not. If a contour plot is necessary, the programmer must specify the low and high DO, BOD and TKN concentrations, representing the minimum and maximum points, respectively, in the concentration ranges to be plotted. The stream segment is expanded by a factor of 2 longitudinally and 4 latitudinally, or each downstream and lateral position (with corresponding concentrations) are repeated two and four times, respectively.

MODEL CAPABILITIES AND LIMITATIONS

As stated previously, TWOD is a steady-state model and does not consider transport and dispersive mechanisms as a function of time. Therefore, the short-term effects of discharging a sewage or stormwater waste stream into an aquatic environment cannot be qualitatively determined.

Since TWOD requires that the location of the surface diffuser be specified and held constant through a particular run, questions arise pertaining to the nature and origin of the waste material being discharged. Essentially, there are two broad classifications used in practice to differentiate between dispersed unmanaged sources from the managed sources having discharges at specific locations. These two classifications are known as non-point and point sources, respectively (Wanielista 1978).

Non-point sources contribute to the impurities found in stormwater runoff and can be classified as being of a rural or urban nature. Thus, when dealing with non-point contribution, stormwater runoff is dealt with exclusively. The quality of non-point generated runoff can be very diverse since areas such as mining, urban roadways, woodlands, construction sites and recreational sites can contribute to the quality of the runoff.

Point sources are typically treated at wastewater treatment plants or managed industrial wastewater plants (Wanielista 1978), though untreated discharges originating at a specific location can be classified in this group. For obvious reasons the quality and quantity of these wastestreams are more uniform in nature than non-point runoffs.

TWOD is designed as a single point source model, therefore, it can analyze wastestreams discharged from sewage treatment plants, industrial (conservative and non-conservative) plant discharges, and from other point sources situated nearby the stream in question. Since this paper is concerned with roadway contributions, this discharge must be assumed to be constant with time.

Non-point contributions can present a particular problem when using TWOD. The transient nature of stormwater runoff make direct input of stormwater hydrographs and pollutographs impossible in TWOD. Attempts to simulate the impact of runoff can be undertaken by estimating the average flowrate and pollutant concentration of the runoff event and using these as input into the model. However, this would probably produce erroneous results since first flush effects are not taken into account. TWOD has the capability of simulating stream impact from diversion system releases. Diversion systems are man-made structures that serve to pre-treat stormwater runoff on a qualitative and to some degree quantitative basis. Thus, they reduce the possibility of flooding conditions and/or shock loading effects on a sewer system or receiving wastewater.

Detention basins are used frequently in pre-treating stormwater runoff from parking lots and roadways before discharging into a receiving water. They minimize the adverse loading effects caused by first-flush effects. Excess runoff is stored in the structure, and then gradually released.
Detention facilities will not reduce the total volume of runoff, but provide redistribution of the rate of runoff over a period of time (Wanielista 1978). Retention basins generally consist of a storage pond or concrete structure

underlined with percolative materials, allowing a fairly constant effluent loading to be indirectly discharged into man-made or natural waterways.

The constant flowrate and pollutant loading provided by the diversion structure can, therefore, be used as input in TWOD. This would then allow the TWOD model to simulate the aquatic impact of discharging stormwater into rivers or streams.

CHAPTER IV

STORMWATER OVERFLOW POLLUTION STREAM MODEL (SWOPS)

INTRODUCTION

If the initial conditions in a stream and/or the rate of pollution load entering this stream vary over time, the solution of interest is the transient one. The transient solution is more complex than the steady-state solution since it is both time and distance dependent. Because of the complexity involved, the technology for treatment of this transient solution has been less well developed.

As a result, the EPA has developed a stream model that considers the transient solution for non-point or stormwater runoff into a stream, or similar natural waterway. The model SWOPS simulates a desired stream segment, and the transient (temporal and spatial) effects of the DO deficit and BOD distribution due to a stormwater discharge. The range of interest for time ranges from time equals 0 to the time at which the solution reaches the steady-state regime.

Most of this chapter is essentially a summary of a portion of the material contained in the EPA 1978 publication, "Stream Models for Calculating Pollutional Effects of Stormwater Runoff." For a detailed explanation of the model used in this research, the reader is directed to the referenced publication.

MODEL DEVELOPMENT

The Stormwater Overflow Pollution Stream Model (SWOPS) is a mathematical model for a natural flowing river or stream developed primarily to study the effect of dispersion within the stream or transient changes in water quality caused by storm and combined sewer overflow events. The stream is assumed to be prismatic, that is, the cross-sectional area and velocity are fixed for all distances along a stream segment.

To develop the mass balance partial differential equations used in SWOPS, the length of river to be studied, or the solution reach, is divided into equal intervals of distance (ΔX) along the stream. The distance intervals represent planes perpendicular to the stream velocity vector. The volumes between the dividing planes are called control volumes or segments.

The governing partial differential equations utilized in SWOPS, under the Lagrange coordinate system, for a particular control volume (2) are given as follows:

BOD:

$$\Delta L_2/\Delta t = Q_{in}L_{in}/V - K_r L_2 + E_c(L_1 - 2L_2 + L_3)/\Delta X^2 \tag{9}$$

Dissolved Oxygen Deficit:

$$\Delta D_2/\Delta t = K_d L_2 - K_a D_2 + E_d(D_1 - 2D_2 + D_3)/\Delta X^2 \tag{10}$$

where:

V	= volume of each increment
Δt	= time increment under study
Q_{in}	= average volume flow from the hydrograph for each Δt
L_{in}	= average BOD concentration from the pollutograph for each Δt
K_r	= rate constant for the loss of BOD by sedimentation and biological activity combined
K_d	= rate constant for the loss of BOD by biological activity alone
K_a	= reaeration constant
E_c, E_d	= dispersion coefficient for BOD and dissolved oxygen, respectively
L_1, L_2, L_3	= mass of BOD in control volumes 1, 2 and 3, respectively
$D_1, D_2 D_3$	= dissolved oxygen deficit in control volumes 1, 2 and 3, respectively

The first term in Equation 9 represents the BOD contribution from a stormwater overflow event divided into Δt segments. The last terms in Equations 9 and 10 represent the net rate of diffusion into control volume 2 for BOD and dissolved oxygen deficit, respectively. These diffusive terms are based on Fick's Law for molecular diffusion.

Equations 9 and 10 represent rate equations for control volume 2, therefore, they are not represented in their general form. They can apply to any control volume, with the exception that if the control volume is not located at an outfall, the term $(Q_{in}L_{in}/V)$ is zero.

When the Lagrange Coordinate System is used, the X = 0 point is fixed with respect to the contents of one of the control volumes at time = 0. Therefore, no advection will occur in stream, since the X = 0 point will travel downstream at the stream velocity.

The model SWOPS is, therefore, a one-dimensional model and is essentially designed for analyzing non-point overflows into aquatic systems. It has the capabilities of examining both transient and steady-state solutions depending on how the pollution load is introduced into the stream, and if it is allowed to simulate conditions after long time periods.

Solution of Equations 9 and 10 are derived by using a numerical integration technique known as the Crank-Nicolson Method. This technique requires the derivation of a linear equation for each point along the X-axis in the solution reach. These linear equations are then solved to advance the solution one time increment.

To implement the Crank-Nicolson Method, the values for L and D in Equations 9 and 10, respectively, can be set equal to the sum of values at the Jth and (J+1)th time points divided by two. This procedure decreases the error introduced by simply taking the values on the Jth time row to estimate values at the (J+1)th time row, a procedure that would require very small distance and time intervals, resulting in large computer time needed to solve the computer linear equation matrix. The Crank-Nicolson Method eliminates this disadvantage. The reader is referred to the corresponding EPA Manual on SWOPS to further clarify the Crank-Nicolson technique and associated mathematical formulations (EPA 1978).

MODEL INPUT REQUIREMENTS

This section introduces and describes the various inputs needed to run SWOPS in its original form. Modifications to SWOPS input format will be discussed in a later section. Refer to Table 4 as an aid in understanding the parameters used as input data in SWOPS. Table 4 provides a chronological listing and description of the input parameters as utilized in SWOPS (modifications included), in addition to the required FORTRAN format specification associated with each parameter. Table 4 may be used as a User's Manual if desired by the programmer.

Essentially, the input can be classified into three distinct categories. These being input associated with:

1. kinetic and hydraulic characteristics of the stream in question

TABLE 4

DESCRIPTION OF INPUT AND OUTPUT PARAMETERS UTILIZED IN SWOPS

INPUT:

Card Group	Parameter	Description	FORTRAN Specification
1	NCASE	Number of data cases to be calculated	I2
2	*ITI	Number of time (ΔT) increments specified for hydrograph and pollutograph input	
	*LDIS1	First distance increment in stream segment containing point of discharge for downstream distance calculations, such that LDIS2 - LDIS1 = 20	3I5
	*LDIS2	Last distance increment in stream segment containing point of discharge for downstream distance calculations, such that LDIS2 - LDIS1 = 20	
3	LIST	Alpha-numeric identification for each data case	40A2
4	KR	Rate constant for removal of BOD by deoxygenation and sedimentation, days^{-1}	
	**KN	Rate constant for removal of ammonia nitrogen by deoxygenation, days^{-1}	
	KA	Rate constant for reaeration from the atmosphere, days^{-1}	

TABLE 4 (Continued)

Card Group	Parameter	Description	FORTRAN Specification
	KD	Rate constant for removal of BOD by deoxygenation, $days^{-1}$	
	***DX	Distance interval for the numerical integration procedure, = velocity*DT (optional)	8F10.0
	DT	Time interval for the numerical integration procedure, hours	
	XN	Number of distance increments in the solution reach	
	TM	Number of time increments to be calculated	
5	EC	Dispersion coefficient for BOD, mi^2/day	
	**EN	Dispersion coefficient for ammonia nitrogen, mi^2/day	
	ED	Dispersion coefficient for dissolved oxygen deficit, mi^2/day	
	VEL	Velocity of the stream, miles/day	
	BEN	Benthic oxygen demand, mg/1/day	
	POINT	Program control: Distance point (number of increments from the first upstream point of the solution reach) at which the storm overflow enters the stream	8F10.0

TABLE 4 (Continued)

Card Group	Parameter	Description	FORTRAN Specification
	Q	Total stream flow, cfs	
	ZM	Program control: Number of time increments in the storm hydrograph	
6	DAY	Program control: Time point at which the program begins printing output, days	
	Y1-Y5	Set concentrations for the BOD or DO increment calculations, mg/l	8F10.0
	DL	Program control: 0 = BOD calculation and 1 = DO calculation	
	PRNT	Program control: 0 = increment calculation printout and 1 = selected calculation printout	
7	X1	Distance increment at which program printout begins	2F10.0
	X2	Distance increment at which program printout stops (note that X2-X1 must equal 20)	
8	QNT(M)	Volume of the storm overflow stream at time increment, M, cfs (M=1, ITI)	
ITI/10 cards ea.	LOT(M)	BOD concentration of the storm overflow stream, mg/l at time increment M	10F8.0 per card

TABLE 4 (Continued)

Card Group	Parameter	Description	FORTRAN Specification
	**LNT(M)	Ammonia nitrogen concentration of the storm overflow stream, mg/l at time increment, M	

*   Program modification
**   Program does not consider ammonia nitrogen oxygen demand in its present form
*** Optional, DX is calculated within the program
**** Card groups 2-8 must be repeated for each data case to be analyzed

OUTPUT: I. PRNT = 0.0 (increment calculation printout)

Parameter	Description
1. M	Time increment of calculation (M=1, TM)
2. N(N1-N2)	Distance increments in solution reach at which program printout occurs
3. *TDAY(M)	Time after introduction of storm overflow stream, hr
4. DOT(N)	DL=1, DO deficit in stream * 100/initial BOD in stream at M and N DL=0, BOD (at N) in stream/initial BOD in stream, at M
5. *DIST(N)	Downstream distance travelled at TDAY(M) for distance increment, N, feet
6. *DOD(N)	DL=0, BOD concentration in stream at M and N, also at TDAY(M) and DIST(N) DL=1, DO deficit in stream at M and N, also at TDAY(M) and DIST(N)
7. *BOD	Initial BOD concentration in the stream without mixing

* Program modification

TABLE 4 (Continued)

OUTPUT: II. PRNT = 1.0 (selected concentration printout

Parameter	Description
1. N	Distance increment along the stream
2. DIST	Distance along the stream at increment N, miles
3. TT	Time of travel downstream, days
4. LO(N)	Carbonaceous oxygen demand (5-day BOD) at N, mg/l
5. *LN(N)	Nitrogen ultimate oxygen demand (exclusive of LO) at N, mg/l
6. DO(N)	Dissolved oxygen deficit at N, mg/l
7. M	Time increment of calculation
8. IT	Time interval at which the maximum BOD or DO occurs
9. TTRAV	Time of travel (I*DT), days
10. DOMAX	Maximum DO, mg/l
11. LOMAX	Maximum LO, mg/l
12. SUM1-SUM5	Sum of DO or BOD at time increment M based on Y1-Y5 inputs
13. IM1-IM5	Number of increments exceeding the Y1-Y5 inputs at time increment M
14. DOT(N)	If DL=0, BOD concentration in the stream at distance interval N/initial BOD concentration in the stream If DL=1, DO deficit in the stream at distance interval N/initial BOD concentration in the stream

* Program does not consider nitrogen ultimate oxygen demand in its present form.

2. the numerical integration scheme (Crank-Nicolson process) and arbitrarily selected parameters (program control) to determine the form of the output, and
3. the hydrological and qualitative characteristics of the storm overflow event.

The parameters in the first category are very important since they determine the extent of longitudinal dispersion of the pollutant due to biological and hydrological action in the stream. Referring to Table 4, the kinetic parameters required as input are the rate constants for: (1) removal of BOD by deoxygenation and sedimentation (KR), (2) removal of ammonia nitrogen by deoxygenation (KN, Optional), (3) reaeration from the atmosphere (KA), and (4) removal of BOD by deoxygenation only (KD).

The rate of turbulent diffusion across the planes which separate the incremental stream volumes must be assessed in the program. The input parameters needed for this are the dispersion coefficient for BOD (EC), ammonia nitrogen (EN, optional), and dissolved oxygen deficit (ED). The rate of oxygen takeup due to sludge deposits on the stream bottom can be estimated by the benthic oxygen demand (BEN).

The hydraulic characteristics of the stream are given by specifying the stream velocity (VEL) and the total stream flow (Q). Since SWOPS is designed to analyze a prismatic stream, these two parameters (as well as all input parameters in categories 1 and 2) remain constant throughout the numerical integration scheme, thus resulting in a constant stream cross-section.

Category 2 contains all the input required to activate the numerical integration process. First, the stream is divided into XN distance increments, each of DX length. The distance POINT at which the storm overflow enters the stream is specified next. The number of time increments (TM) in addition to the time interval (DT) required for the numerical integration procedure must be specified. These parameters are very important since they effect computer time requirements and model accuracy.

The program control parameters will now be explained. SWOPS has the versatility of investigating one or more stormwater events successively for constant or variable stream conditions. The variable NCASE allows this flexibility as described in Table 4. In addition the stream segment of interest in the solution reach is investigated by specifying the parameters X1 and X2. The entire channel can be analyzed by varying X1 and X2 (by segments of twenty distance intervals) downstream between distance intervals 0 to XN.

The number of time increments (ZM) in the storm hydrograph is also inputed. The significance of this parameter will be discussed later.

Program output control parameters also serve as input in SWOPS. These parameters are described in Table 4, and include the variables DAY, Y1-Y5, DL, PRNT. These will be discussed in the SWOPS Output Section.

Category 3 contains the input specifying the hydrograph and pollutograph of the storm overflow event. Since the storm overflow is transient in nature, the parameter, ZM, has to be specified as noted earlier. The event is described by defining the:

1. Volume (hydrograph) of the storm overflow event at time increment M (QNT(M)).
2. BOD concentration of the storm at time increment M (LOT(M)).
3. Ammonia nitrogen concentration of the storm overflow stream at time increment M (LN(M), optional)

In its original form, SWOPS allowed a maximum of 30 time increments to describe the hydrograph and pollutograph. Modification to SWOPS has now made it possible to describe more than 30 time increments (refer to the chapter on Modifications Incorporated in SWOPS). The magnitude of the volume and BOD concentrations of the storm can be variable (transient) or constant (steady--state, square pulse) in nature.

Even though SWOPS was designed to accept kinetic and qualitative input for stream response to ammonia nitrogen loading, the input is optional. The development of SWOPS was not advanced to consider the ultimate oxygen demand, on stream dissolved oxygen levels, resulting from ammonia nitrogen loadings due to stormwater overflow events. This situation can be corrected by the addition of appropriate terms to the governing dissolved oxygen deficit and BOD material balance equations for a stream.

Another possibility in estimating nitrogen ultimate oxygen demand is to account for nitrogenous biochemical oxygen demand (NBOD) when inputing BOD concentrations associated with a stormwater overflow event. This would allow the examination of stream response to carbonaceous and nitrogenous BOD loadings.

MODEL OUTPUT REQUIREMENTS

The purpose of this section is to present a concise explanation of the output generated by SWOPS before modifications were incorporated in the program. This will then allow a more complete understanding of why revisions to the input and output formats in SWOPS are justifiable. Chapter V discusses these modifications in detail.

There are two distinct output formats possible in SWOPS depending on the value assigned to PRNT (see Table 4). The first variation is the increment calculation printout, while the second is referred to as the selected concentration printout.

In order to study the impact of a stormwater event on aquatic life, the EPA initiated research in this area. SWOPS was, therefore, made more flexible by having the capability of presenting its results in increment calculation form. Essentially, the increment calculation format presents the maximum DO or BOD, time of travel, and time interval at which the maximum BOD or DO occurs. These results can also be determined using the selected concentration format though not as easily, since the results are not grouped in this manner. In addition, the number of increments (distance intervals) exceeding arbitrarily selected BOD or DO concentrations (Y1-Y5) are calculated and illustrated for every time increment calculated. This allows the determination of the time period and distance downstream that stream water quality is possibly detrimental to fish and other aquatic organisms. The input Y1-Y5 should then be selected as the BOD or DO concentration limits, which can be harmful to aquatic organisms.

Unfortunately, the interest in this area decreased with time, resulting in abandonment by the EPA. The selected concentrations calculation results were not verified and certain phases were not advanced properly. Therefore, this part of SWOPS should not be used until further research and verification is completed.

The second variation in output is the selected concentration printout. The EPA conducted numerous test runs and verified them with the closed form Streeter-Phelps Model at steady-state conditions (EPA 1978). It can, therefore, be concluded that the increment concentration approach is satisfactory for determining the transient impact of a stormwater overflow event on a stream.

Basically, the program output listed all the input information discussed in the preceding section. The results of the numerical integration procedure were then presented in a dimensionless matrix. The matrix results (BOD or dissolved oxygen deficit) were listed as dimensionless characters as a function of the time increment (M) in question, and the distance intervals X1 to X2 in the solution reach. The entire stream reach can be analyzed by varying X1 to X2 from 0 to XN, respectively.

The BOD and dissolved oxygen deficit concentrations were presented as dimensionless ratios to initial stream BOD for clarity in tracing the pulse downstream. This allowed the maximum concentrations in the stream to be located quickly.

Since the results were shown as dimensionless characters, difficulty in interpreting them as real physical information can arise, especially for a person not proficient in the numerical integration technique. In order to make SWOPS readily understandable for stream dispersion and, therefore, mixing zone analysis, revisions to the output representation were in order. In Chapter V, revisions made in SWOPS to achieve this criteria are reviewed.

CHAPTER V

MODIFICATIONS INCORPORATED INTO SWOPS

In this section, modifications to SWOPS as they relate to program input and output capabilities are made. Particular attention is directed to those modifications incorporating mathematical formulations.

INPUT MODIFICATIONS

Referring to Table 4, three new input parameters are included in SWOPS. These parameters are ITI, LDIS1 and LDIS2. To enable the programmer to subdivide a hydrograph or pollutograph into more than 30 time increments, if necessary, the parameter ITI was included. ITI is the number of time (Δt) increments specified for hydrograph and pollutograph input. Though research conducted in this area (refer to Chapter VI) has shown that all hydrographs and pollutographs incorporated can be approximated with 30 or less time increments, it was felt that this flexibility should be left to the programmer's discretion. It should be pointed out that even though the numerical analysis efficiency will increase as the size of the time increments decrease, more computer time and, therefore, higher programming costs will be incurred as a result. However, computer time and cost requirements were found to be minimal for all runs on the computer utilized during the present research.

To calculate downstream distances at each time increment calculated, the parameters LDIS1 and LDIS2 are used. LDIS1 and LDIS2 are the first and last distance increment in the stream reach containing the point where the stormwater overflow event enters the stream. These characters differ from X1 and X2 in that they are constant, while the latter can vary depending on the stream reach segment being analyzed. LDIS1 is equal to X1 and LDIS2 is equal to X2 only when the stream segment (or distance matrix) being investigated originally contained the discharge point. This approach always allows the programmer to follow the pulse or profile downstream in its entirety without losing track of it. This procedure is consistent with the Lagrange Coordinate System and will be discussed further in the output modifications section.

OUTPUT MODIFICATIONS

Most of the output modifications involved rearranging the old output format to one simpler to understand and applicable to the present research. All time and distance results are presented in units of hours and feet, respectively. The short duration of the storm overflow events used in the current investigation required analysis to be conducted in hourly time units as well as distance requirements to be measured in feet. Therefore, all FORTRAN statements incorporating time and distance computations, within the source program, have been accordingly revised.

Instead of presenting the results as a dimensionless time and distance matrix, as was originally done, the results are now presented in block form combining the old BOD and DO deficit ratios with the predicted concentrations for "real" time and distance measurements after the stormwater overflow event enters the stream in question. Every concentration downstream occurs at a specific time and distance downstream, thus simulating actual physical data in the stream. The old format is left intact to preserve its integrity and allow the user to relate the matrix approach to the new one. The arrays containing the predicted BOD and DO deficit concentrations are calculated within the source program and simply printed.

To calculate the time and distance downstream where a predicted BOD or DO deficit concentration occurs, it was necessary to formulate the appropriate equations relating these units to the LaGrange Coordinate scheme. If the Euler Coordinate System is used, the pollutional pulse will travel downstream at a velocity equal to $\Delta X/\Delta t$, where ΔX and Δt are, respectively, the size of the distance and time increments used. Since this coordinate system is stationary, the pulse will eventually be lost, unless the number of distance increments in the solution reach is large enough to allow an adequate number of stream segments to be modeled. This is physically impossible to do with any program as a result of the large number of increments and computer time that would be needed.

As mentioned earlier to conserve the physical depiction of the pollutional pulse, SWOPS incorporates the LaGrange Coordinate System. This scheme allows the coordinate system to move downstream at the velocity of the stream. Therefore, the pulse can easily be examined with time and distance for the effects of dispersion. The process is analogous to a photograph being taken of the pulse at every time increment calculated.

The equations derived represent the physical time and distance traveled by the coordinate system at a specific lattice point DX, a point in the X direction and DT, a point in the time direction from another point. Refer to Table 4 for parameter identification. For a time increment of calculation M = 1, ..., MT, the starting time, STIM, increases by DT. Therefore, the time period elapsed in each iteration is represented by the following equation:

$$TDAY(M) = STIM * 24 \tag{11}$$

STIM is multiplied by 24 to change the units from days to hours. The character TDAY(M) physically represents the time elapsed, in hours, after introduction of the first time increment loading in the stormwater event. It is printed with each time increment calculated in the results.

A distance character T(N) is introduced as follows:

$$T(N) = (VEL*TDAY(M)*2)/(LDIS1 + LDIS2) \tag{12}$$

The ratio (LDIS1 + LDIS2)/2 represents the distance increment where the stormwater event begins to enter the stream reach. Therefore, for each distance increment N = 1..., and time increment M calculated, T(N) represents the distance downstream that the coordinate system has moved at time TDAY(M). This distance character allows SWOPS the capability to follow the pulse downstream.

The downstream distance, DIST(N), for a particular lattice point NA(N) can then be calculated by the following equation:

$$DIST(N) = NA(N)*T(N)*5280 \tag{13}$$

DIST(N) is presented in the modified output along with its corresponding distance increment NA(N). The velocity used in the calculations has been adjusted to an hourly rate. The hourly time units proved beneficial when investigating the stormwater overflow events utilized in this research since detrimental impacts of these storms proved to be short-lived in most cases.

The initial BOD concentrations imparted to the river by the storm runoff are calculated by SWOPS in its original form. Since the initial BOD in the stream is assumed to be zero in this one-dimensional model, the imparted BOD represents the concentration resulting from the mixing of the first mass and volumetric loading increment of the event, with the control volume located at the point of discharge. The control volume has a volumetric flow equal to that of the stream. This mixing does not consider dispersion contributions since it occurs instantaneously at time equal to zero. This measured concentration gives the user an indication of the initial strength of the discharge at the outfall, thus enabling him to assess the volumetric mixing capabilities of the stream. This initial BOD is also printed in the modified output.

CHAPTER VI

RESULTS AND DISCUSSION OF MIXING ZONES IN RIVERS AND STREAMS

The results discussed in this chapter are presented in two section, these being: (1) Mixing Zone Curve Development Using TWOD and (2) Research Conducted with SWOPS. Each section contains the results followed by a discussion of the results.

MIXING ZONE CURVE DEVELOPMENT USING TWOD

To illustrate the practical application of site specific data in accessing mixing zone requirements using TWOD, mixing zone curves were derived for a wastestream of varying strength discharged into a hypothetical stream. Due to a lack of field data available, the stream simulated contained similar biokinetic and qualitative parameters as the stream utilized by the developers of TWOD (Florida DER 1979).

The stream was modeled as a rectangular channel consisting of 25 longitudinal and 21 latitudinal distance increments representing 220 and 2.50 ft, respectively. The first longitudinal and latitudinal increments represent the starting point in the reach, or X = 0.0 and Y = 0.0, respectively. The stream reach was, therefore, a mile in length and 50 ft in width. The average depth assigned to each lateral distance increment was allowed to vary from two feet at the bank diffuser to 5 feet near the center and finally 0.10 feet at the opposite bank. This resulted in a cross-sectional area of approximately 140 square feet. A variable depth in the cross-section is typical of most existing streams. A stream bank diffuser of 5 feet in length was chosen. The following parameters were held constant in each computer run:

1. Initial Stream Conditions

   a. BOD = 0.0 mg/l
   b. TKN = 0.0 mg/l
   c. Temperature = 30°C
   d. DO saturation constant, C_s = 7.40 mg/l

2. Stream Biokinetic Parameters

a. Reaeration rate, KA = 1.54 $days^{-1}$

b. BOD oxidation rate, KR = 0.30 day^{-1}

c. BOD oxidation and settling rate, KD = 0.30 day^{-1}

d. TKN oxidation rate, KN = 0.10 day^{-1}

Mixing zone length curves were then derived by individually varying either the initial DO concentration of the receiving water or the BOD and DO concentrations of the effluent wastestream. TWOD mixing zone requirements were then observed by varying the flowrate of the effluent or receiving water, while maintaining the other constant, to determine what combinations or dilution ratios satisfied the mixing zone length standard of 800 meters. The desired points are then plotted on an effluent flowrate and mass loading (as lb/day BOD) versus receiving water flowrate coordinate system, consequently producing the desired curve.

The mass loadings include both carbonaceous and nitrogenous BOD. Since both induce an oxygen demand on stream resources, it was felt necessary to include the combined effect. A factor of 4.57 multiplied by the total oxidizable nitrogen (TKN) was used in the analysis to determine the oxygen demand for 1 mg/l TKN (Thomann 1972). The TKN loading (TKN_e) for the effluent was assumed to be 10 mg/l for all TWOD runs.

A total of eight mixing zone curves for varying effluent and receiving water quality were derived during this phase of research. Figure 1 represents the decision process incorporated in order to produce the desired curve by illustrating all possible effluent (DO_e and BOD_5) and receiving water (DO_0 and BOD_0) water quality combinations used in the analysis. Only four of the curves (Figures 2-5) are represented here for brevity since these are sufficient for the proceeding discussion.

By superimposing one mixing zone length curve over another, the impact of a particular effluent wastestream or stream water quality parameter on the regions of non-acceptance and acceptance for mixing zone length requirements can be determined. For instance, superimposing Figure 3 on Figure 2 reveals that an increase in the initial DO concentration of the stream increased the acceptable mixing zone region for plant operation. Therefore, the mixing zone length requirements can be met at higher effluent flowrates and pollutant loadings since dilution can occur readily at low stream flow conditions.

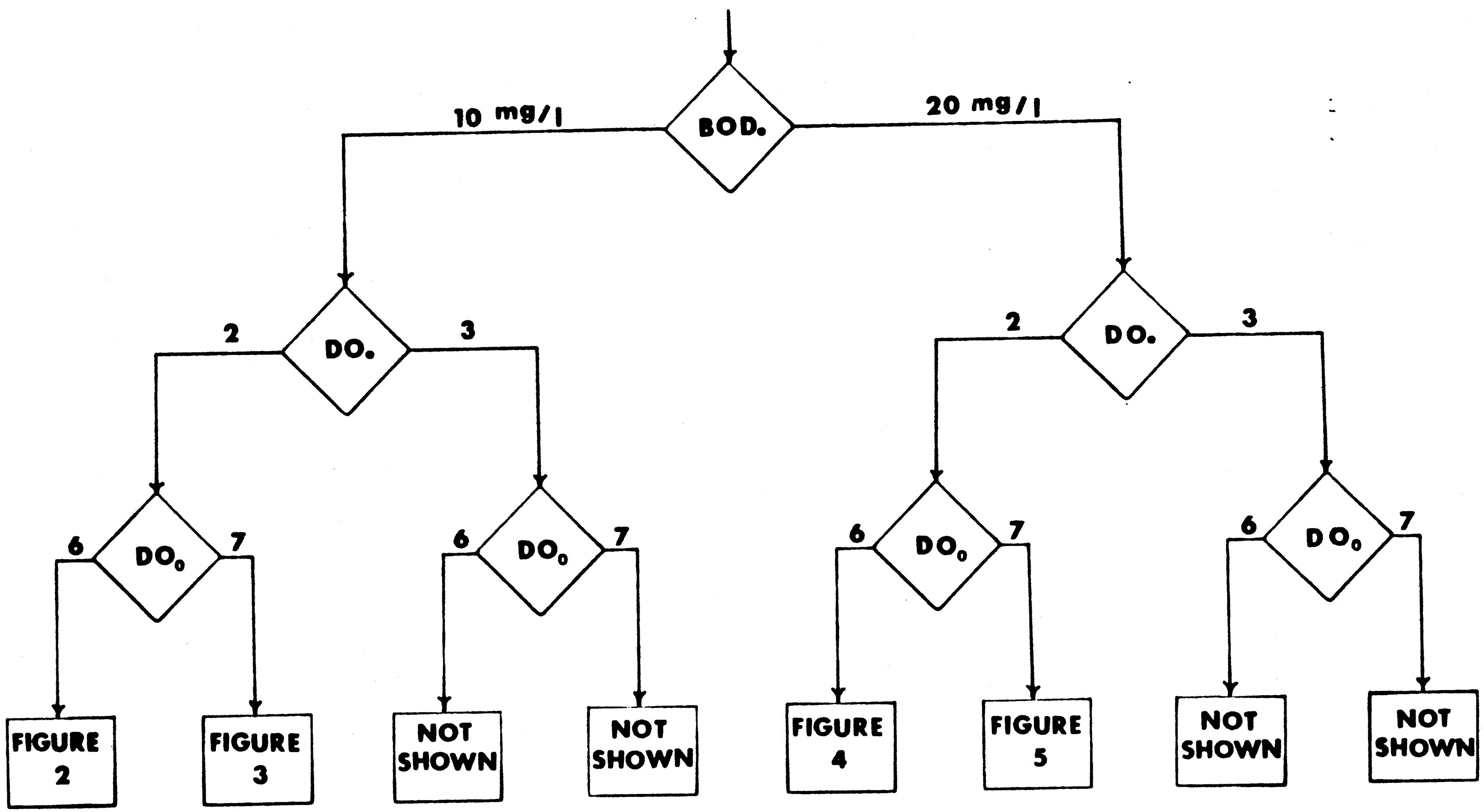

Fig. 1. Decision process utilized for mixing zone length curves development.

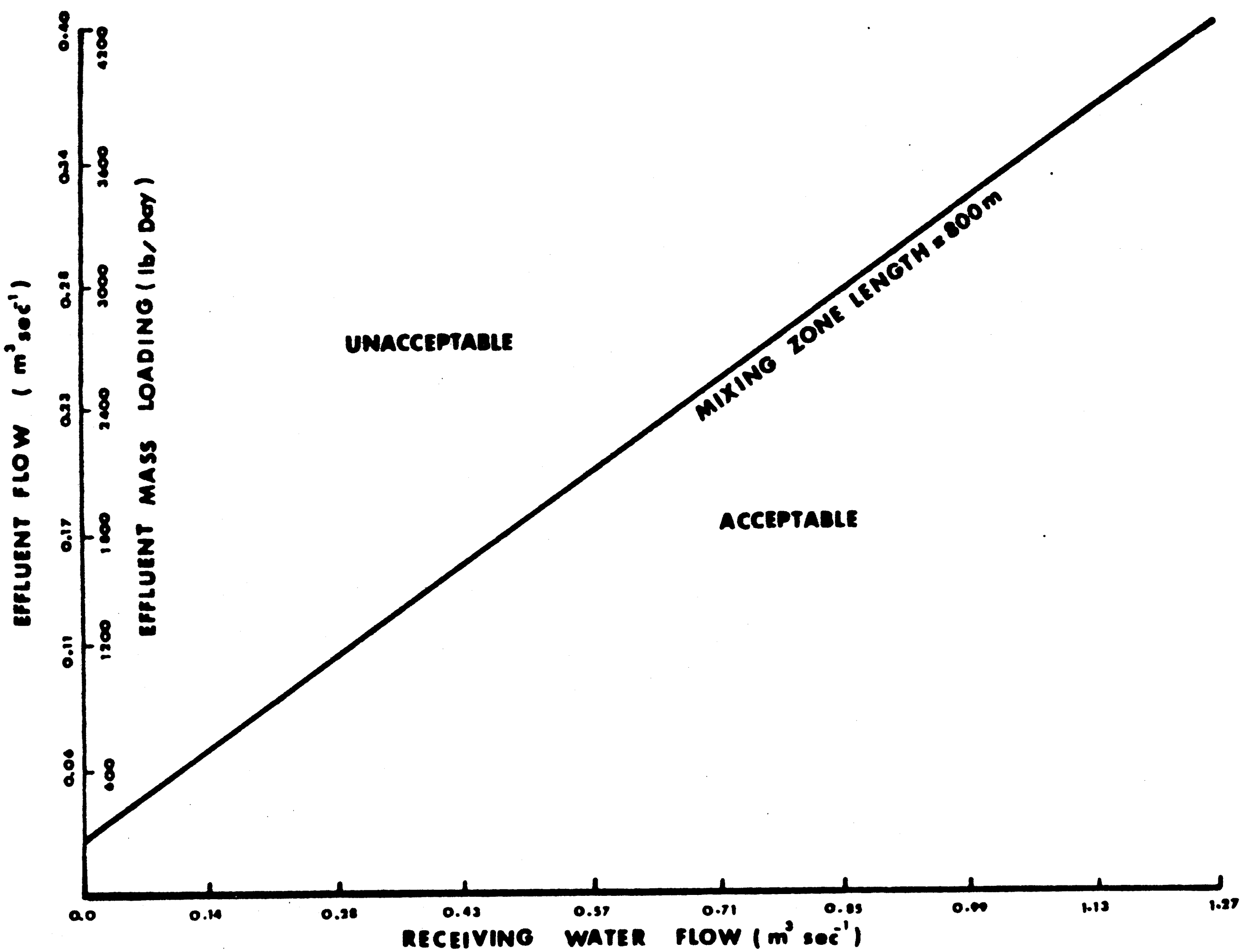

Fig. 2. Mixing zone length curve derived for a hypothetical stream; assuming DO_o = 6 mg/1, DO_e = 2 mg/1 and BOD_e = 10 mg/1.

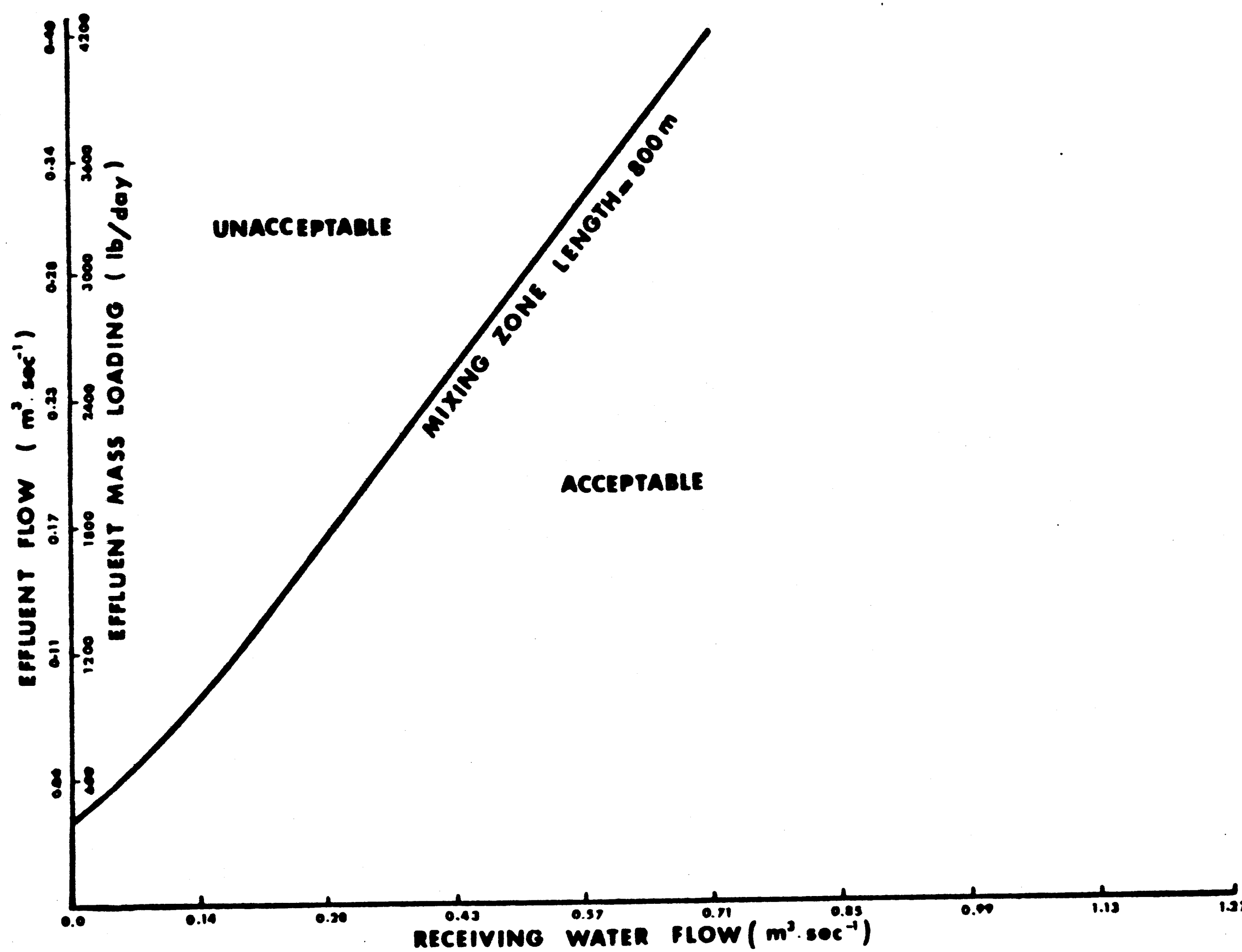

Fig. 3. Mixing zone length curve derived for a hypothetical stream; assuming DO_o = 7 mg/1, DO_e = 2 mg/1 and BOD_e = 10 mg/1.

Similarly, higher effluent wastestream DO concentrations produce the same effect since the improved quality enhances the assimilative capacity of the stream in question, provided that all other conditions remain constant.

Increasing the mass loading (BOD and TKN) of the effluent requires higher receiving water flowrate and/or greater lateral dispersion capabilities to transport the pollutant and meet the same mixing zone requirements for a weaker strength effluent. This is evident when Figures 4 and 5 are compared to Figures 2 and 3, respectively. Increasing the effluent BOD loading in these two cases results in the shifting of the curve downward or directly decreasing the region of acceptable plant operations. For a given stream flowrate, the plant operator would have to decrease the effluent flowrate or loading discharged into the stream to meet the 800 m standard. The higher BOD loadings further deplete the oxygen resources in the stream, resulting in the implementation of stormwater management or sewage treatment controls.

If the stream's water quality (DO, BOD and TKN concentrations) was to deteriorate due to commercial or municipal discharges upstream of the plant in question, the mixing zone curves would also shift downward, resulting in the curtailing of effluent discharges or loadings in this facility. It should be noted that present Florida regulations require a cumulative mixing zone length of not greater than 10 percent of the total length of the stream in question (Florida Administrative Code Chapter 17-4). However, TWOD does not have the capability to model cumulative mixing zones in its present form.

Increasing the instream length of the bank diffuser would also minimize mixing zone length requirements since a larger segment of the stream's cross-section would be utilized for mass transfer. The reader is referred to the DER report on TWOD for the impact of varying bank diffuser length on mixing zone requirements (Florida DER 1979).

When TWOD was used to model a wastestream with a DO concentration of 5 mg/l, for DO_e and BOD_e concentration ranges of 6-7 mg/l and 10-20 mg/l, respectively, no mixing zone was required in the stream as expected. This occurred because the predicted concentrations of DO never fell below the stream water quality standard of 5 mg/l as required by the Florida Administrative Code in Chapter 17-3.

Increasing the lateral dispersion coefficient in these test runs would shift the mixing curves upward, therefore, providing larger regions of

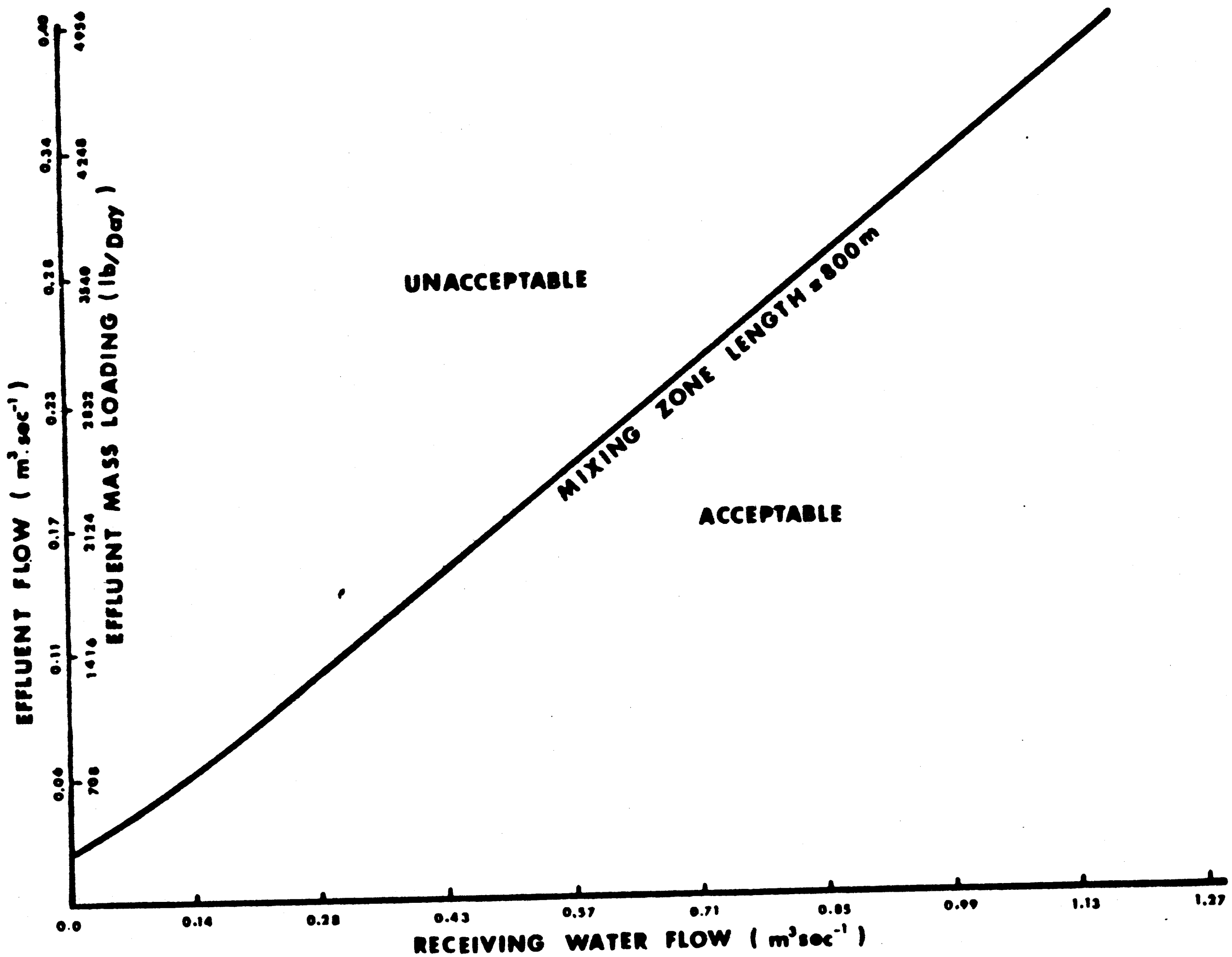

Fig. 5. Mixing zone length curve derived for a hypothetical stream; assuming DO_o = 7 mg/1, DO_e = 2 mg/1 and BOD_e = 20 mg/1.

acceptable operations. Since plume dispersion in the lateral direction is enhanced, a larger effective stream volume would be needed to dilute the concentrated waste.

Impacts due to other stream parameters on mixing zone curves are summarized below:

1. Increasing reaeration coefficients will shift the curve upwards.

2. Increasing KR, KD and KN will shift the curve downward since biological activity and, therefore, oxygen consumption is increased.

These impacts serve to inform the user on what to expect if mixing zone curves are to be derived for different streams.

Since TWOD is a two-dimensional model, mixing zone cross-sectional as well as surface area requirements can be analyzed for a point source effluent discharge into an existing stream. An attempt was made to model these area requirements as a function of the dilution ratios encountered by varying point source effluent flowrates into a receiving water of constant biokinetic, geometric and hydraulic characteristics. The dilution ratio being defined as the ratio of effluent volumetric discharge to receiving water volumetric flowrate.

The hypothetical stream modeled is the same one modeled in the mixing zone length curve development except that the initial DO concentration (DO_0) was held constant at 6 mg/l. The effluent wastewater qualitative characteristics being defined by:

$$DO_e = 2.0 \text{ mg/l}$$

$$BOD_e = TKN_e = 10.0 \text{ mg/l}$$

which was one of the loadings examined in the previous analysis. A stream diffuser of 5.0 feet length was again utilized.

To obtain reasonable results either the receiving water or the wastewater effluent flowrate must be kept constant, while the other one is allowed to vary. This would prevent the possible existence of more than one area requirement for a particular dilution ratio if both flowrates are allowed to vary. The dispersion potential in the lateral direction, as defined in TWOD is a function of water depth and mean stream velocity; therefore, multiple area requirements are possible for one dilution ratio since the stream flowrate can vary along with the effluent flowrate, producing varying magnitudes of lateral dispersion. This effect can be corrected by holding the receiving flowrates constant or by assuming constant depths in the cross-section, therefore, producing relatively constant dispersion coefficients through the test runs. The former approach was chosen since it is more applicable in natural streams.

The mixing zone surface and cross-sectional area curves were developed for two constant receiving water flowrates. These being 14.0 and 25.24 cfs, respectively. These two conditions are analogous to those experienced in the field for dry and wet season flow. Therefore, it is possible to examine the change in mixing zone area requirements due to these flowrates for constant wastestream quantitative and qualitative characteristics. Acceptable and unacceptable regions for treatment facility operations can then be examined based on the dilution ratio.

Figure 6 contains the surface area mixing zone requirements as calculated by TWOD. A normalized zone surface area curve is shown for the two receiving water flowrates investigated. Normalization was based as a percentage of the total surface area for the stream reach. As expected, the surface area requirements for the mixing zone increases as the dilution ratio increases for both curves. An increasing dilution ratio implies that the large wastestream discharges increases the pollutant (BOD and TKN) loadings into the stream, thus impeding the assimilative capacity of the stream. This results in a greater mass transfer downstream as well as in the lateral direction due to advective and dispersive mechanisms, thereby increasing the surface area requirement for the mixing zone. This effect is more prominent for a stream flowrate of 14.0 cfs than at 25.0 cfs since the lower receiving water flowrate does not have the dilution capabilities of the 25.0 cfs flowrate. Therefore, as the wastewater flowrate increases, the initial mixing (diluting) potential is minimized, resulting in higher predicted DO concentrations downstream and, hence, larger mixing zone surface area requirements. Similarly, if the quality of the wastestream is allowed to further deteriorate, the surface area curve would shift upwards for both receiving water flowrate conditions.

Both curves in Figure 6 suggest that there exists a dilution ratio range where the rate of increase in surface area requirements decrease for increasing dilution ratio. These ranges are for dilution ratios between 0.23-0.43 and greater than 0.83 for the 25.24 cfs stream flowrate curve. This produces the S-shaped curve, suggesting that the assimilative capacity of the stream

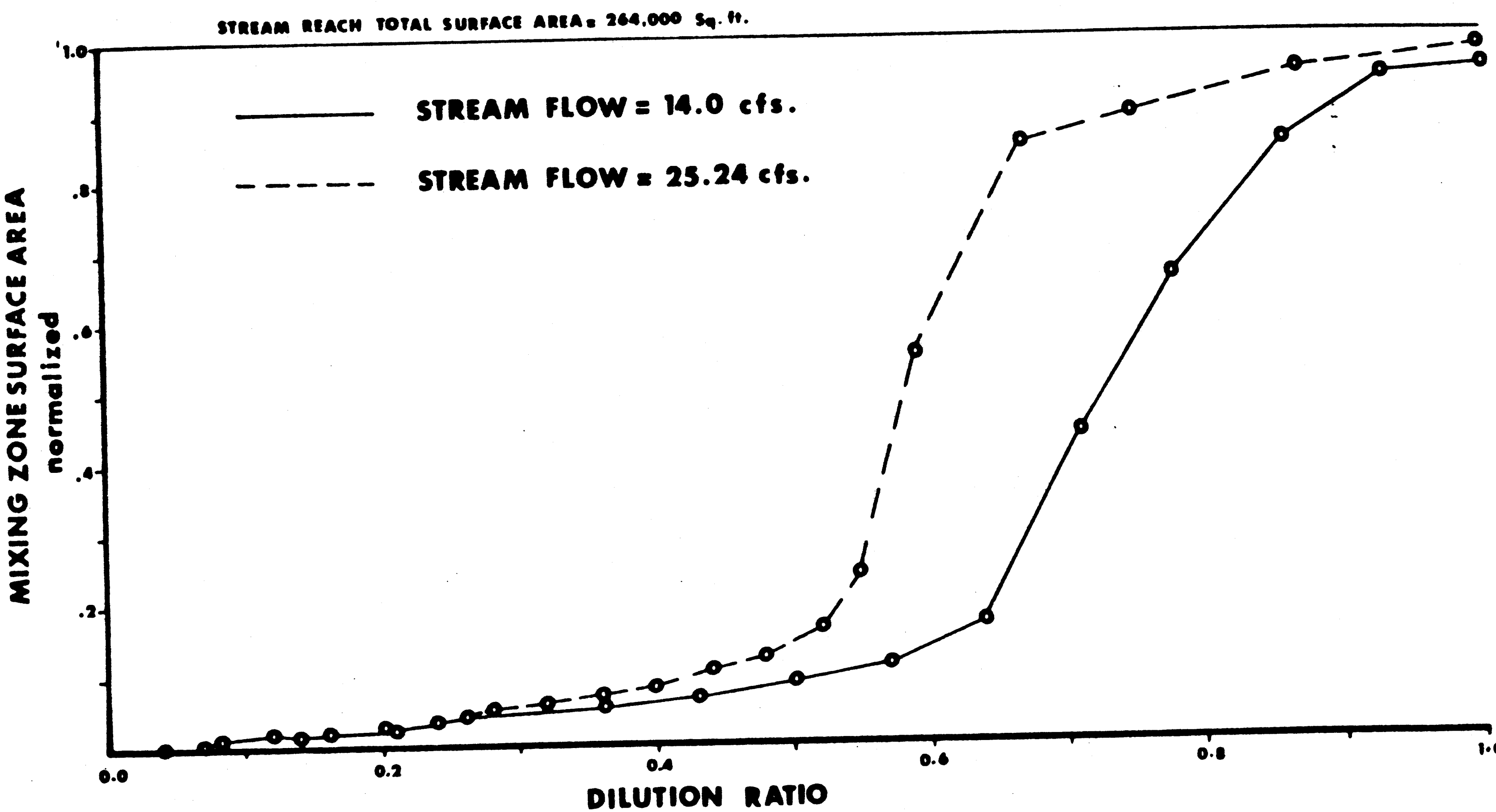

Fig. 6. Mixing zone surface area curve derived for a hypothetical stream; assuming DO_o = 6 mg/1, DO_e = 2 mg/1 and BOD_e = 10 mg/1.

increases sufficiently to handle the increased loadings experienced in this region. The characteristic shape is more pronounced for the 25.0 cfs curve than for the 14.0 cfs curve since the initial dilution potential in the outfall area is greater in the former. As the dilution ratio increases, the surface area requirements for the mixing zone approaches the total stream reach cross-sectional area of 264,000 square feet, as expected in both curves. The large momentum associated with the diffuser at this high effluent flowrate disperses the pollutant for roughly the entire cross-section of the stream. After this complete mixing, the advective mass transfer will carry the pollutants downstream through the entire length of the stream reach.

To illustrate the cross-sectional mixing zone requirements as a function of dilution ratio for the same stream flow and wastestream qualitative and quantitative conditions, Figure 7 is presented. The results shown were from the same test runs conducted with TWOD for the mixing zone surface area analysis. Again, the curves were normalized to represent a percentage of the total cross-sectional area for the stream reach. The curves were developed to see if compliance with the 25 percent cross-sectional area regulation, discussed in Chapter I, exists for the 14.0 and 25.0 cfs stream flowrates utilized before. Since the hypothetical stream was calculated to have a constant cross-sectional area of approximately 140 square feet, a dilution ratio producing a mixing zone cross-sectional area greater than 35 square feet would result in unacceptable mixing zone requirements.

Again, the results in Figure 7 suggest that cross-section area requirements for mixing zones increase as a function of increasing dilution ratio. This results because of the increased mass transfer in the lateral direction due to increased diffuser jet momentum associated with the higher discharge flowrates. The transfer of pollutants in the lateral direction, thus, is more dominant initially than that due to advection, resulting in a large cross-sectional mixing zone region. This phenomenon is more pronounced in the 14.0 cfs curve than the 25.0 cfs curve due to the smaller advective mass transfer potential.

As observed in Figure 7, the cross-sectional area standard of 25 percent of the total stream cross-section is exceeded in both cases for dilution ratios greater than approximately 0.25. Treatment facilities operating below this point will at least comply with this requirement. If operating conditions exceed a dilution ratio of 0.25, the wastestream flowrate would have to

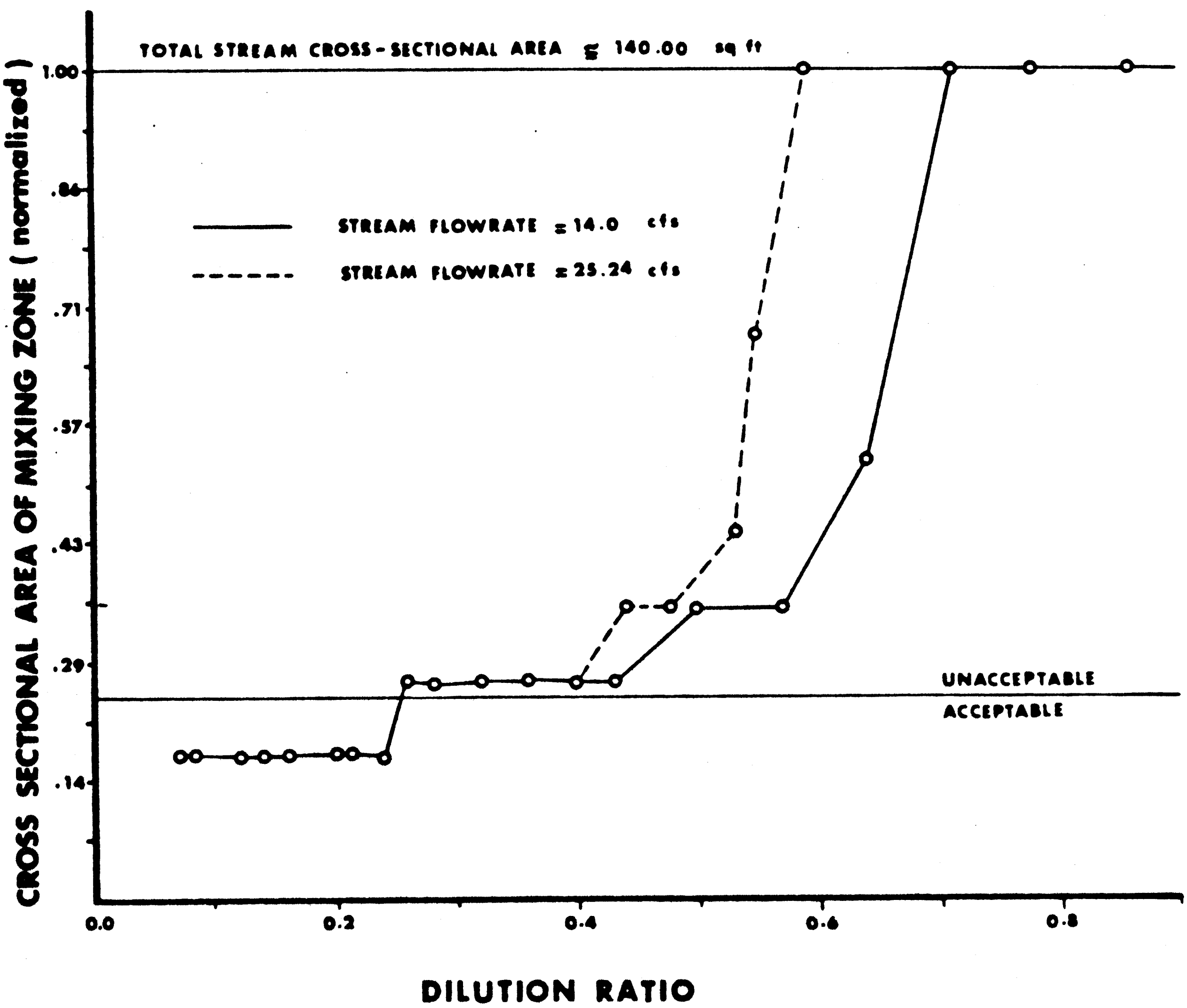

Fig. 7. Mixing zone cross-sectional area curve derived for a hypothetical stream; assuming DO_o = 6 mg/l, DO_e = 2 mg/l and BOD_e = 10 mg/l.

either be decreased or more efficiently treated for pollutant removal. Higher quality wastestreams would shift this curve downward, resulting in a larger range of acceptable operating conditions. As the dilution ratio increases, the mixing zone cross-sectional area required approaches the total cross-sectional area of 140 square feet of the stream. The large momentum jet associated with these dilution ratios spread the pollutant as far as the opposite bank because of the small receiving water flows.

The large dilution ratio regions requiring constant cross-sectional area requirements are a result of the TWOD numerical analysis. Since the model can only predict BOD, DO and TKN concentrations at points across and downstream, separated by a specified distance increment, large portions of the reach can be neglected if the magnitude of these increments are set too high. This resulted in the constant cross-sectional area regions experienced in Figure 7. If the lateral increment is specified as larger than 2.5 feet, these constant lines would be even more prevalent. Therefore, the magnitude should be specified as small as possible in order to arrive at reasonable results.

If the diffuser length is increased, a larger stream area can be effectively utilized for pollutant mass transfer. It would then be reasonable to conclude that both the mixing zone cross-sectional and surface area curves would shift upwards at constant stream and wastestream conditions. As mentioned earlier, increasing the diffuser length would effectively decrease the mixing zone length requirements. Therefore, a trade-off between area and length requirements could be utilized provided that current mixing zone standards are complied with.

For a dilution ratio of 0.25, an effluent flowrate equal to 6.31 cfs (0.18 m^3/sec) with a stream flowrate of 25.24 cfs (0.71 m^3/sec), Figure 2 illustrates that this operational point requires a mixing zone length of less than 800 m, thus lying in the acceptable region of operation. Therefore, this operating point satisfies both mixing zone length and cross-sectional area standards. The same applies for the 0.25 or effluent flowrate equal to 3.5 cfs (0.1 m^3/sec) for a stream flow equal to 14.02 cfs (0.40 m^3/sec). However, this is not always the case as has been shown by TWOD in the past. At very high stream and low wastestream flowrates the mixing zone length standard of 800 m can be exceeded due to strong advective forces while the cross-section area requirements comply with the 25 percent recommendation.

The mixing zone length and cross-sectional area curves should be incorporated to study the overall effect of stream response to mixing zone requirements.

If the average depth in the stream was assumed to be constant, a direct relationship between mixing zone cross-sectional and surface area requirements would exist. It would then be possible to model mixing zone requirements on a volumetric basis. The complete analysis would then require mixing zone length, area and volume analysis for a particular stream in question.

Since it is possible to satisfy the mixing zone cross-sectional area recommendation and not satisfy the mixing zone length requirement of 800 m for a particular discharge capacity, a mixing zone volume concept should be utilized in developing appropriate mixing zone standards. For instance, the 800 m length standard and the 25 percent cross-sectional area recommendation can be combined to develop a mixing zone volume standard, generally applicable to any existing stream or river. The volume standard can represent a percentage of the total volume of the stream reach in question, therefore, permitting the mixing zone length and cross-sectional area constraints of 800 m and 25 percent, respectively, to be applied directly.

The surface area requirements for a mixing zone can also be incorporated in this approach. This would allow a three-dimensional view of the mixing zone since the surface area as well as the cross-sectional area requirements of the plume can, therefore, be modeled. Again, a surface area mixing zone standard should be based as a percentage of the total surface area of the stream reach.

Standards based on a mixing zone volume concept would allow tighter restrictions on point and non-point source discharges into a stream. The mixing zone length standard of 800 m as currently regulated cannot prevent serious impairment to a stream's ecosystem since it does not represent a true three-dimensional picture.

This portion of the research conducted on mixing zone length and area requirements is in line with the recommendations set forth by Stefan and Gulliver (1978) and Paily (1981). They suggested using site specific data in developing relationships between mixing zone geometric requirements and the dilution ratios as well as other stream hydraulic characteristics.

The mixing curves developed illustrate the practical usefulness of TWOD. Mixing zone curves such as these can be derived for existing wastewater (point and "controlled" non-point) discharges into natural waterways. The curves can serve as a management tool for the engineer or plant operator in meeting existing or future mixing zone regulations. These curves are applicable under steady-state conditions; to analyze the short-term impact of wastestreams on mixing zone requirements a transient model should be used.

TIME SENSITIVITY ANALYSIS WITH SWOPS

The predicted concentration of a pollutant downstream will vary with the number of time increments specified in approximating known hydrograph and pollutograph input when utilizing time sensitive (transient), analytical or numerical models. That is, as the number of time increments specified for analysis are increased, the predicted concentration (as a function of time and distance) at a point downstream should approach some constant level.

The purpose of this time sensitivity analysis was to develop a graphical relationship between hydrograph shape and the number of time increments required to adequately access stream response using SWOPS. This would allow the user to apply the technique for a known hydrograph (or pollutograph) overflowing into an existing stream.

Hydrograph (or pollutograph) shape characteristics can be adequately described by the following three parameters: (1) peak discharge, or time to reach peak discharge, (2) the first moment of the hydrograph which defines the mean, and (3) the second moment about the mean which represents the variance (hence standard deviation) of the hydrograph. Therefore, for a given hydrograph, if it is possible to describe its time to peak, first and second moments, it is possible to determine an appropriate number of time increments needed in SWOPS to assess stream impact to achieve a desired (or adequate) model accuracy.

The first and second moment of a hydrograph would also describe the shape characteristics of its pollutographs (BOD or TKN) since all are interdependent with similar times of generation, peak and termination. The first and second moments can be found by using standard statistical equations applicable to sampling distributions (Viessman, Knapp, Lewis and Harbaugh 1977 and Haan 1977).

These three shape characteristics can be grouped as a dimensionless ratio defined as follows:

$$\frac{\text{2nd Moment of Hydrograph } (t^2)}{\text{Time elapsed to peak } (t) * \text{1st Moment of Hydrograph } (t)} \qquad (14)$$

where:

t = time unit used in the analysis

A relationship between the number of time increments specified for a hydrograph (and pollutograph) and this dimensionless ratio is, therefore, possible to achieve desired SWOPS accuracies for a hypothetical stream with constant geometrical, hydraulic and biokinetic parameters.

The hypothetical stream, referring to Table 4, used in this analysis is characterized by the following:

1. Stream Geometrical Characteristics
   a. XN = 100.0
   b. Cross-sectional area = 125.0 ft^2
   c. Point = 50.0
2. Stream Hydraulic Characteristics
   a. Flowrate = 5.0 ft^3/sec
   b. Velocity = 0.6545 mile/day
3. Stream Longitudinal Dispersive Characteristics
   a. EC = EC = 0.01 mi^2/day
4. Stream Biokinetic Characteristics
   a. KR = KR = 0.30 day^{-1}
   b. BEN = 0.0 mg/1/day

In addition, the reaeration coefficient, KA, was set equal to 1.54 day^{-1} as with the stream modeled with TWOD. The flowrate and velocity of the stream are quite small since it was observed that SWOPS response to the stormwater events used in this analysis was minor, due to the weak strength of most of these storms at higher flowrates. The low flowrate and velocity used allowed measurable response of BOD concentrations but not DO deficit concentrations. Therefore, to avoid "fixing" the pollutographs to insure measurable DO deficit concentrations, the predicted BOD concentrations were used in approximating SWOPS response accuracies. By observing the peak BOD concentration occurs downstream, the user can, therefore, approximate where the critical DO deficit will occur since it is dependent on the peak BOD concentration, as well as on the biokinetic rate constants existing for the stream in question.

There were four stormwater events used in the study. Two of the storms utilized are shown in Tables 5 and 6, respectively. The third event is representative of an overflow from a combined (sewage and stormwater) sewer system.

TABLE 5

0.75 HOUR STORMWATER EVENT UTILIZED FOR TIME SENSITIVITY ANALYSIS ON SWOPS

Sample Time (pm)	pH	COND (µmho/cm)	ALK (mg/l) $CaCO_3$	TURB (NTU)	SS (mg/l)	VSS (mg/l)	BOD_5 (mg/l)	Carbon (mg/l)				Nitrogen (mg/l-N)				Phosphorus (mg/l-P)		Σ Rain (in)	Avg. Flow (cfs)
								IC		TOC		TKN		NH_3-N	NO_3-N				
								Total	Diss.	Total	Diss.	Total	Diss.			Total	Diss.		
2:10	7.3	70	22	34	292	122	17.7	12.8	4.4	53.9	31.9	2.00	1.14	0.05	0.92	0.34	0.07	0.04	1.6
2:15	7.2	40	14	32	174	41	6.2	10.0	3.9	40.6	14.6	0.80	0.66	0.05	0.32	0.29	0.02	0.09	3.3
2:18	7.3	30	16	13	74	16	2.6	6.7	4.4	14.7	5.1	0.72	0.60	0.05	0.20	0.28	0.18	0.22	7.9
2:23	7.3	26	10	7	13	7	2.1	4.2	3.9	7.7	4.4	0.77	----	0.07	0.10	0.26	0.13	0.34	13.5
2:26	7.3	26	14	6	8	4	2.3	4.2	4.7	5.9	4.2	0.82	0.43	0.07	0.12	0.34	0.21	0.37	12.9
2:28	7.3	27	14	5	8	7	2.0	3.9	3.6	6.8	5.3	1.12	0.42	0.12	0.13	0.30	0.14	0.39	11.9
2:30	7.3	26	14	5	8	8	1.8	4.4	5.0	9.9	5.7	0.72	----	0.05	0.20	0.36	0.16	0.43	8.3
2:35	7.5	55	16	12	14	9	2.0	7.2	7.2	11.8	9.5	0.82	0.67	0.05	0.34	0.48	0.14		5.5
AVG.	7.3	37.5	15	14	74	27	4.6	6.7	4.6	18.9	10.1	0.97	0.65	0.64	0.29	0.33	0.13		

SOURCE: U.S. Environmental Protection Agency. Stormwater Management to Improve Lake Water Quality, January 1981.

TABLE 6

1.83 HOUR STORMWATER EVENT UTILIZED FOR TIME SENSITIVITY ANALYSIS ON SWOPS

Sample Time (pm)	pH	COND (μmho/cm)	ALK (mg/l) $CaCO_3$	TURB (NTU)	SS (mg/l)	VSS (mg/l)	BOD_5 (mg/l)	Carbon (mg/l)				Nitrogen (mg/l-N)				Phosphorus (mg/l-P)		Σ Rain (in)	Avg. Flow (cfs)
								IC		TOC		TKN		NH_3-N	NO_3-N				
								Total	Diss.	Total	Diss.	Total	Diss.			Total	Diss.		
3:15	7.7	120	525	54	336.0	172.5	8.3	22.7	10.9	60.1	23.9	2.26	.92	0.06	1.00	0.53	0.07	0.20	2.8
3:30	7.6	70	24	42	132.0	81.0	8.8	8.6	5.0	21.9	18.2	1.37	.82	0.16	0.56	0.37	0.08	0.30	4.2
3:45	7.5	60	295	44	265.0	135.0	8.0	12.7	6.4	43.6	9.5	1.38	.68	0.13	0.46	0.26	0.10	0.60	5.6
3:55	7.5	50	21	40	128.5	76.5	5.9	9.5	4.5	4.9	3.0	0.78	.51	0.08	0.29	0.48	0.12	1.60	23.0
4:10	8.0	70	30	18	16.5	16.5	2.6	9.1	6.4	7.3	5.9	1.00	.45	0.07	0.14	0.14	0.13	3.20	10.0
4:15	7.5	50	27	14	13.5	13.5	2.9	9.1	6.4	6.5	3.7	0.75	.48	0.06	0.14	0.13	0.11		7.9
4:55	7.3	40	13	6	12.5	-----	3.1	3.6	3.2	8.1	6.9	0.95	.71	0.05	0.35	0.15	0.10		0.2
AVG.	7.6	66	134	31	129.1	82.5	5.7	11.8	6.1	21.8	10.2	1.21	.65	0.09	0.42	0.29	0.10		

SOURCE: U.S. Environmental Protection Agency. <u>Stormwater Management to Improve Lake Water Quality</u>, January 1981.

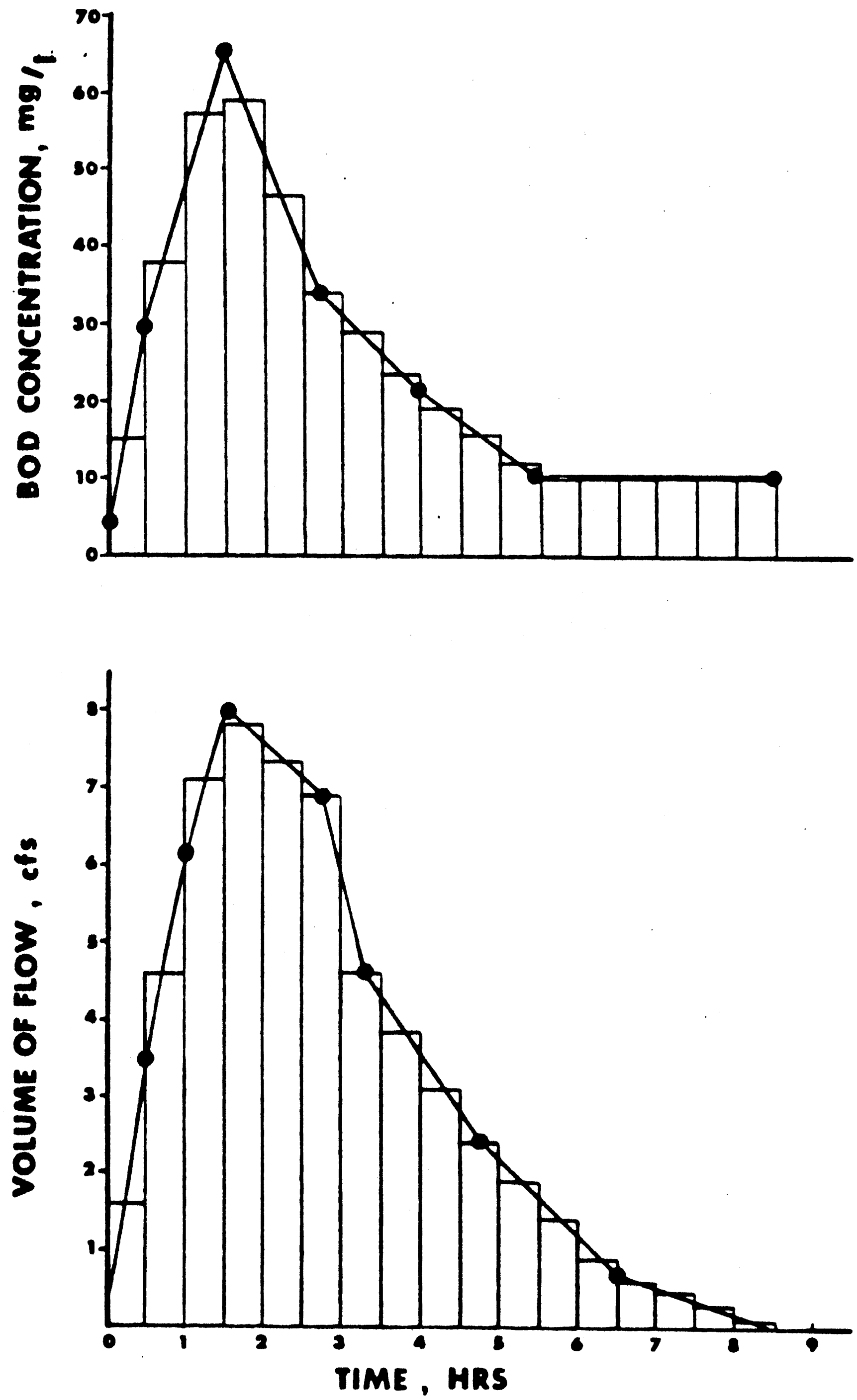

Fig. 8. Typical 8.5 hour combined overflow event divided into 30 minute time increment for SWOPS time sensitivity analysis.

SOURCE: U.S. Environmental Protection Agency. STREAM Models for Calculating Pollutional Effects of Stormwater Runoff, August 1978.

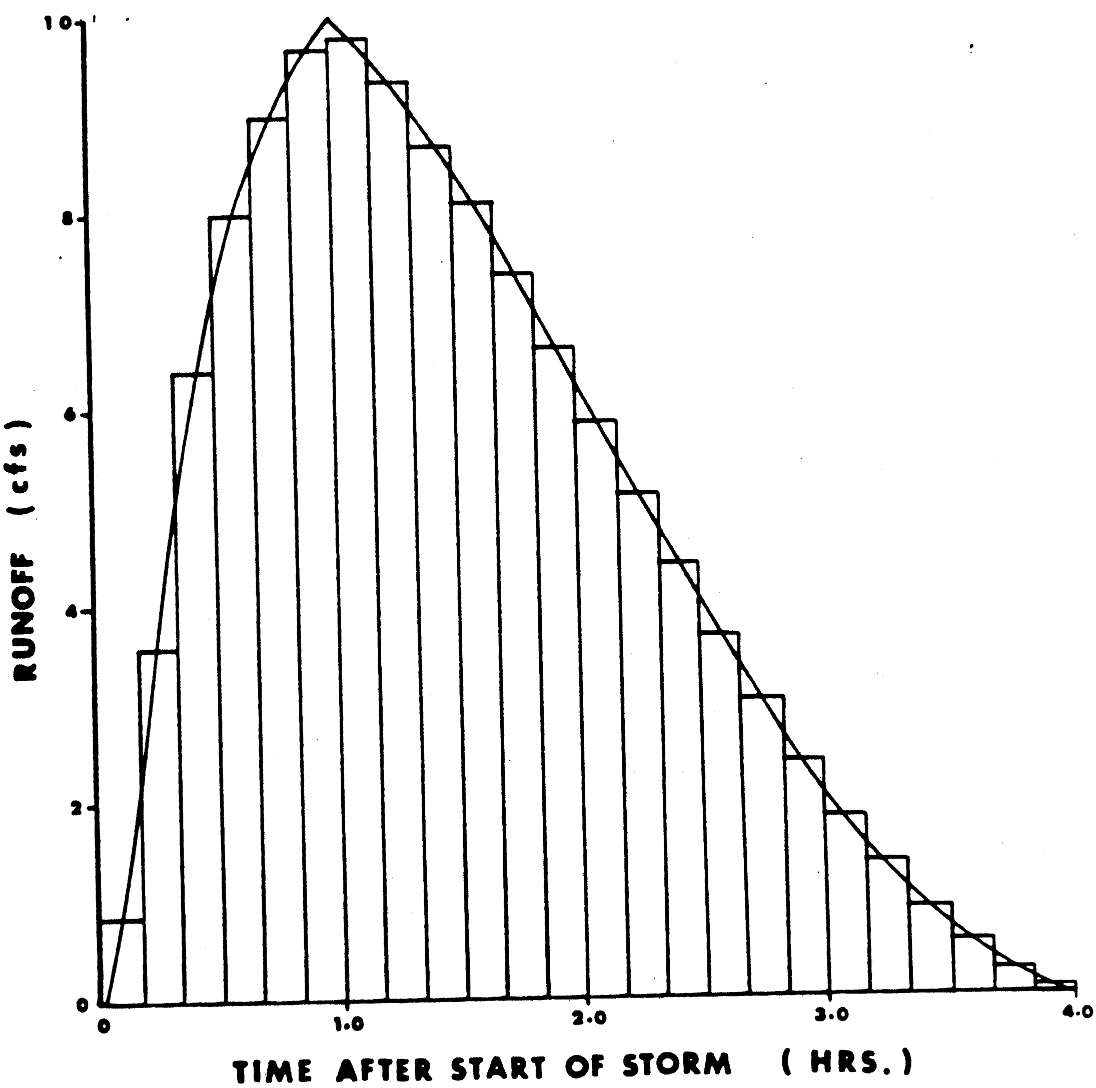

Fig. 9. Hydrograph associated with the 4.0 hour stormwater event divided into 10 minute time increments for SWOPS time sensitivity analysis.

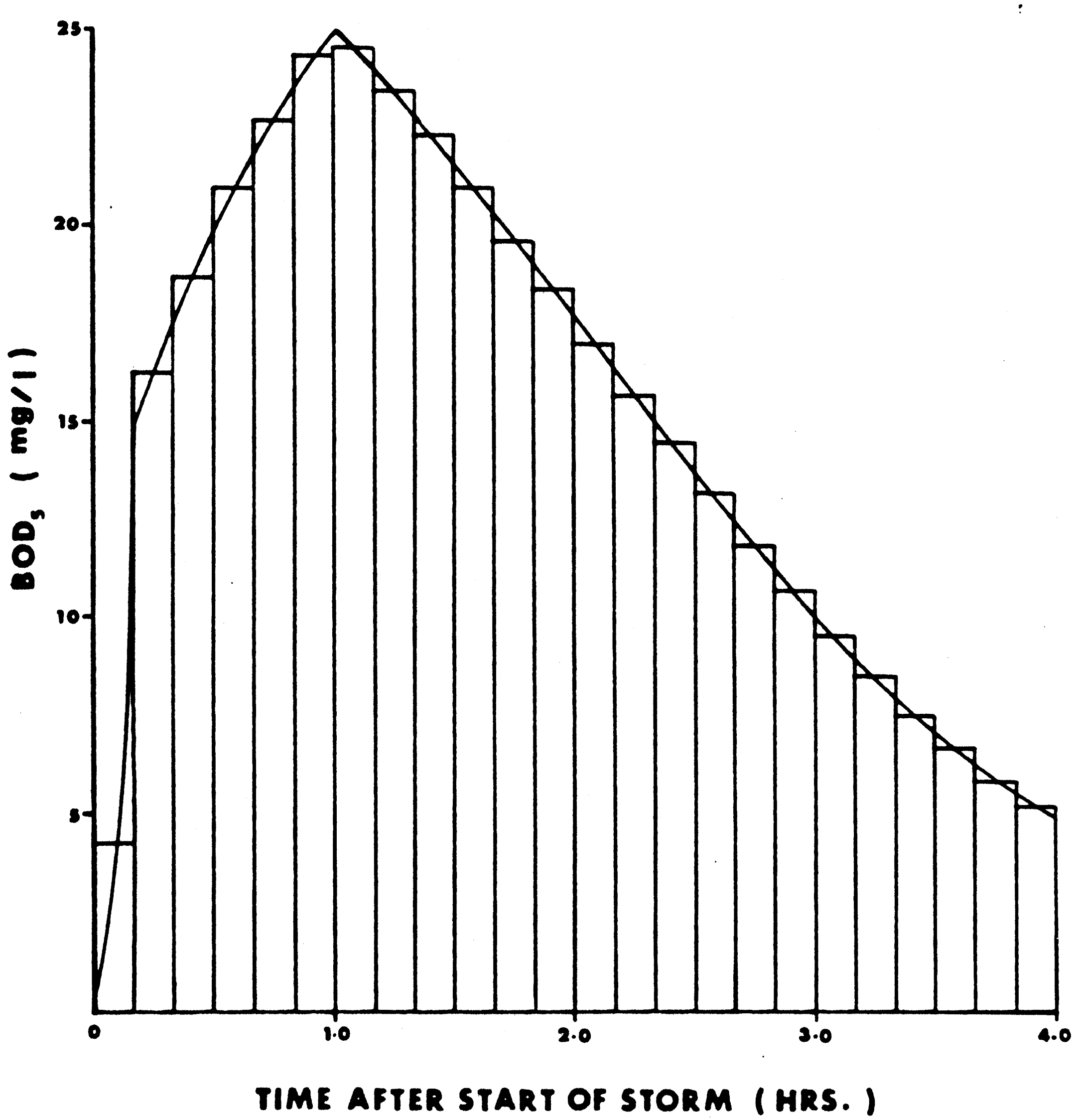

Fig. 10. Carbonaceous BOD_5 pollutograph associated with the 4.0 hour stormwater event divided into 10 minute time increments for SWOPS time sensitivity analysis.

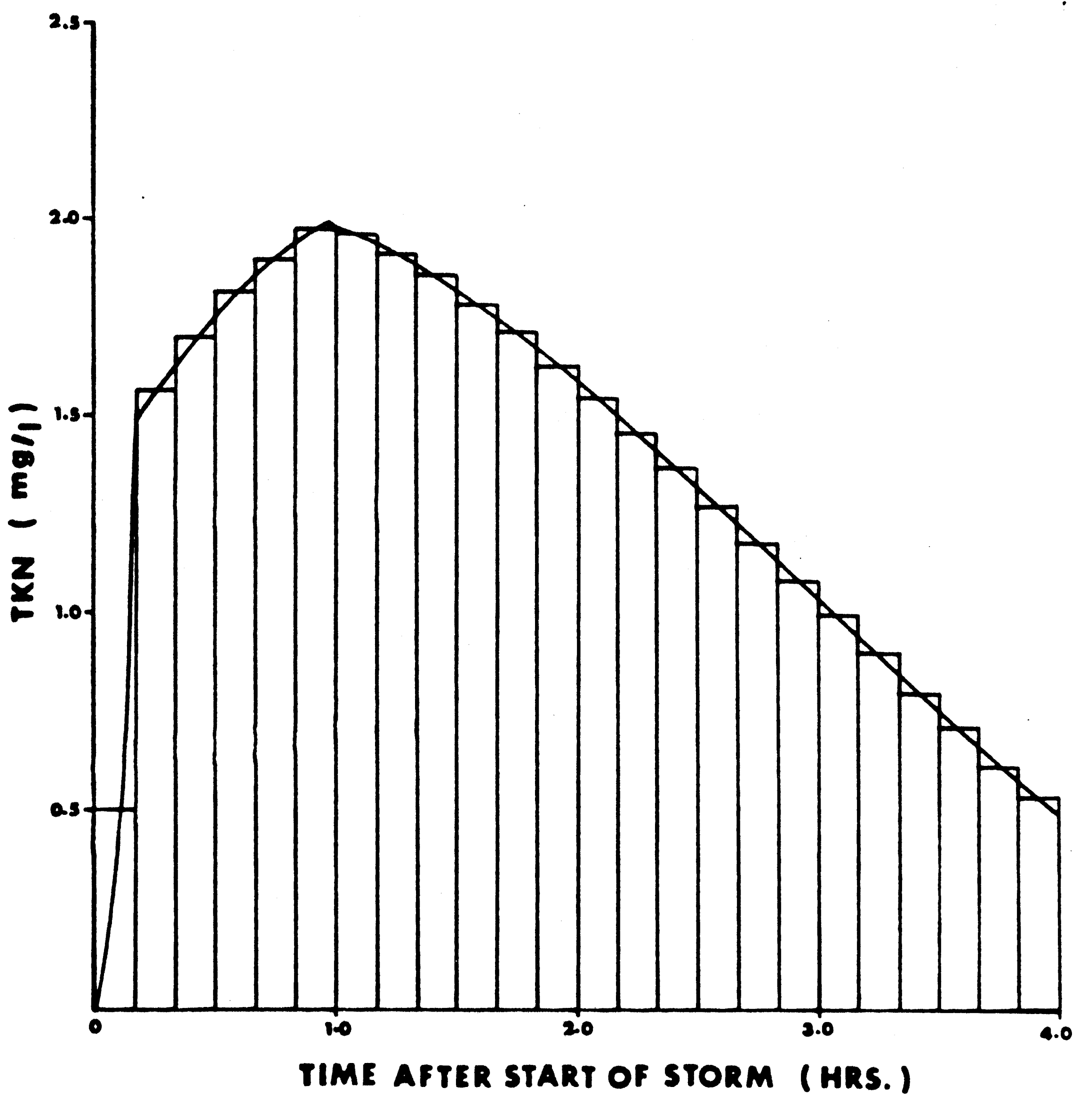

Fig. 11. Total Kjeldahl nitrogen pollutograph associated with the 4.0 hour stormwater event divided into 10 minute time increments for SWOPS time sensitivity analysis.

events described in Tables 5 and 6 were divided by varying time increments of 5, 6, 9, 10 and 15 minutes. (These events divided into 5, 10 and 15 minute increments are shown in Figures 12-20 as an illustrative example). The results of computer runs (SWOPS) for the 6, 9, 10 and 15 minute time increments were then compared to the results of the 5 minute run by comparing the peak BOD concentration predicted at 0.5 and 1.0 hours after introduction of the storm event into the stream for the 0.75 and 1.83 hour storms, respectively. The variations of the predicted peak concentration from the 5 minute incrementation, were then observed for the 6, 9, 10 and 15 increment results at these particular time points. The variations were measured as an accuracy percentage of the 5 minute peak concentrations. The ratio of the absolute difference between the peak concentrations at 5 minute incrementation and the increment size in question, and the largest peak concentration observed for the two increment sizes subtracted from 1, defines the accuracy.

A similar procedure was utilized in determining SWOPS accuracies for the 4 hour and 8.5 hour storm events. However, the test runs were conducted for 10, 15, 20 and 30 minute time increment specifications for the 4 hour storm, while 20, 30, 45, 60 and 90 minute increments were used in the 8.5 hour storm. The first size increment specified was used as the base in both storms to observe variations in predicted peak BOD concentrations.

As expected, the accuracy of SWOPS in predicting the peak BOD concentration decreases as the number of time increments specified in approximating the stormwater or combined event decreases. Observations for all four stormwater overflow events utilized suggested that the percent error encountered in using the computer model increases dramatically (to infinity) as the number of time increments specified approaches zero. This is a result of the numerical integration techniques incorporated in SWOPS, incapability to converge on the desired solution (or concentration) at very large time and distance field requirements. Therefore, the solution fails to approach a constant concentration level.

The results of these sensitivity runs are shown in Table 7. The dimensionless shape ratios calculated for each storm are also shown. The product of the number of time increments specified with the corresponding size of of the increment should approximate the duration of the stormwater overflow event. However, exact agreement is not always possible since SWOPS requires time increments of uniform size.

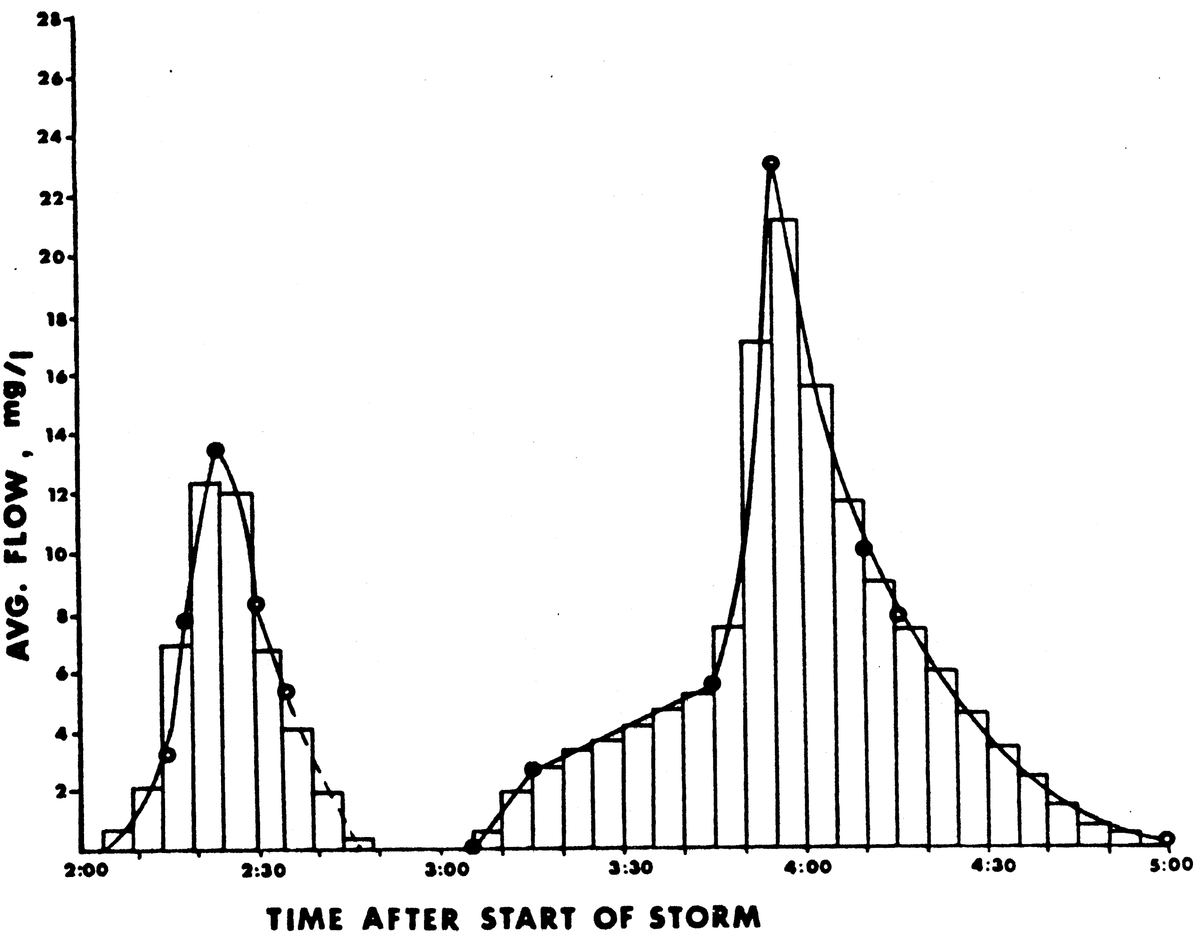

Fig. 12. Hydrographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 5 minute time increments for SWOPS time sensitivity analysis.

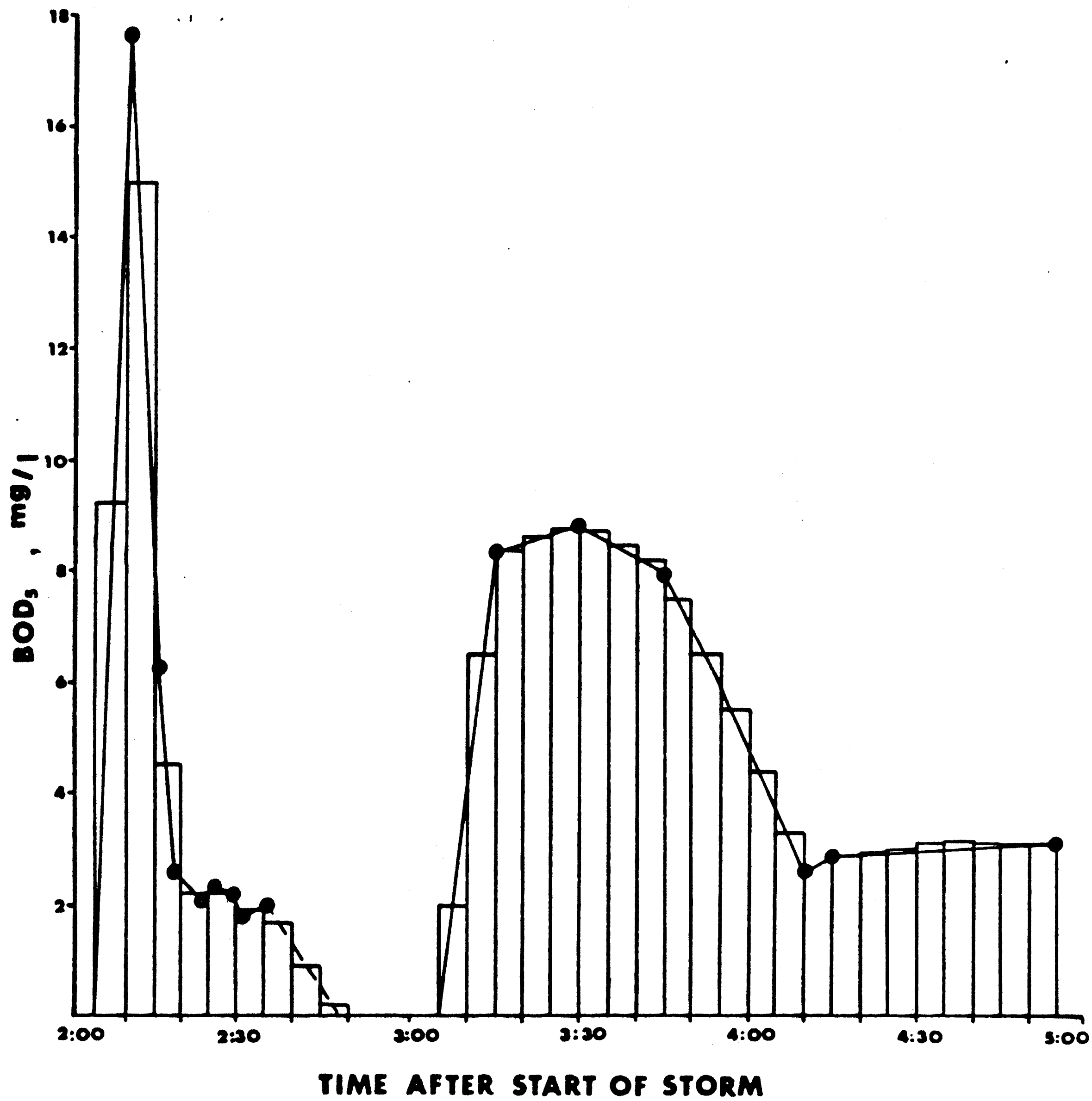

Fig. 13. Carbonaceous BOD_5 pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 5 minute time increments for SWOPS time sensitivity analysis.

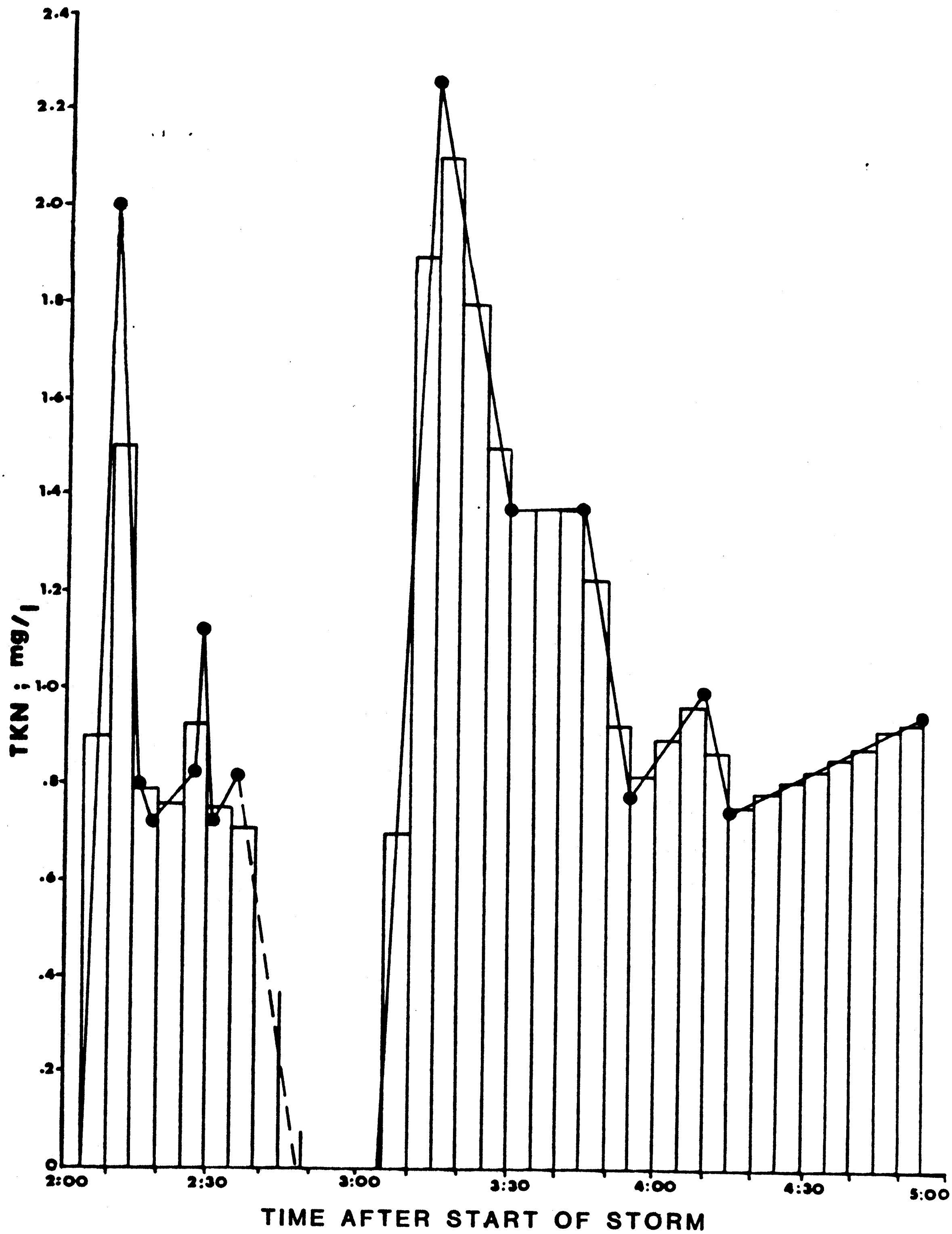

Fig. 14. Total Kjeldahl nitrogen pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 5 minute time increments for SWOPS time sensitivity analysis.

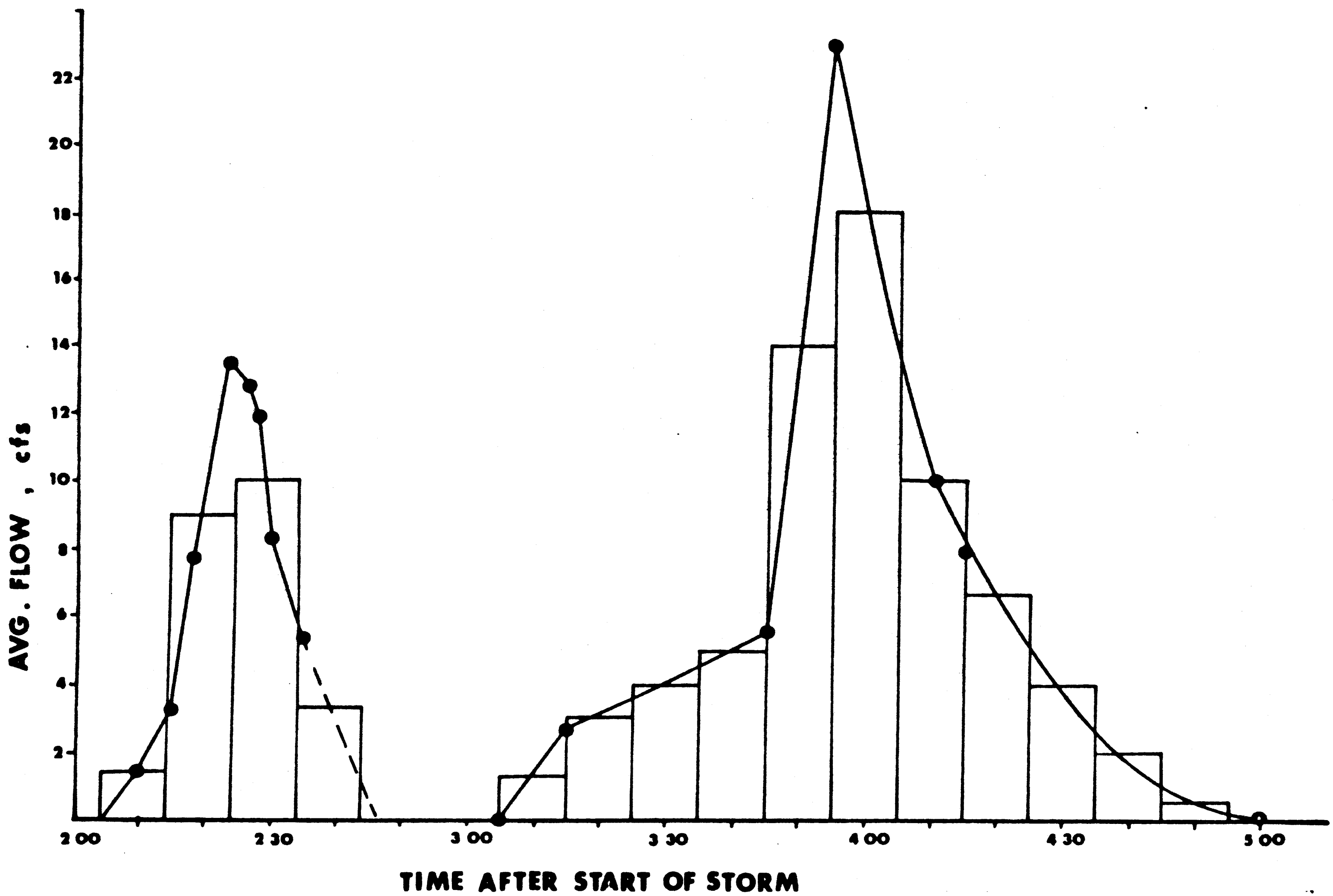

Fig. 15. Hydrographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 10 minute time increments for SWOPS time sensitivity analysis.

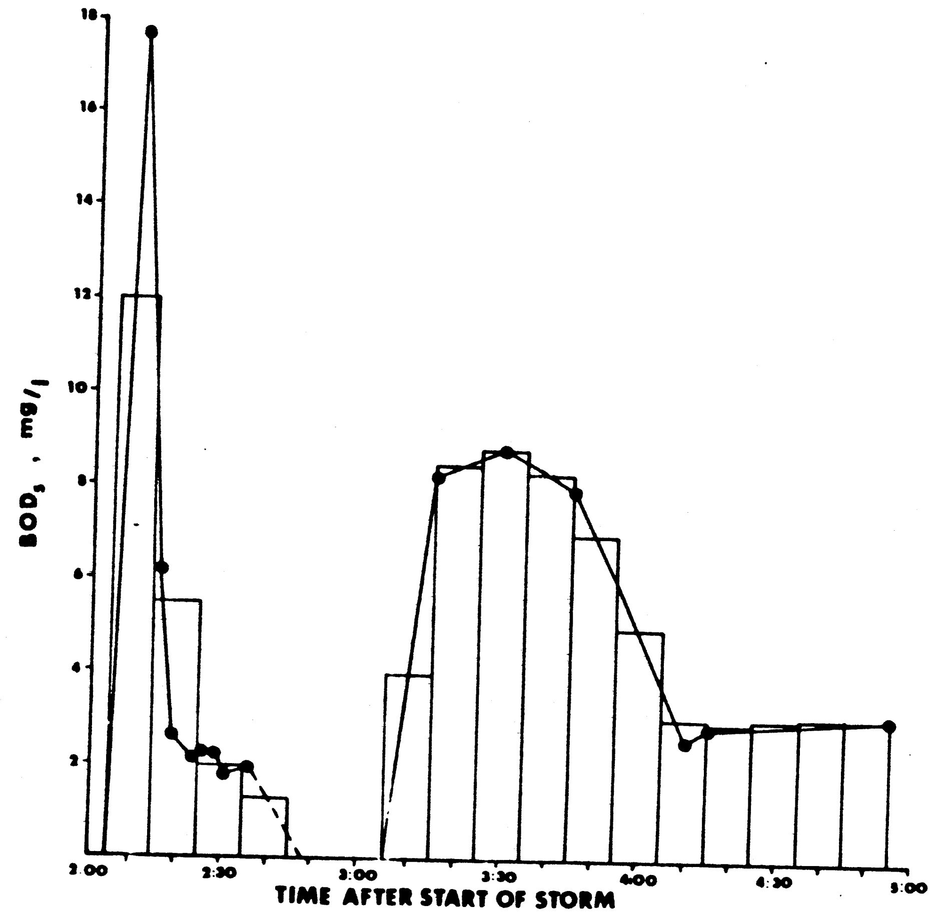

Fig. 16. Carbonaceous BOD_5 pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 10 minute time increments for SWOPS time sensitivity analysis.

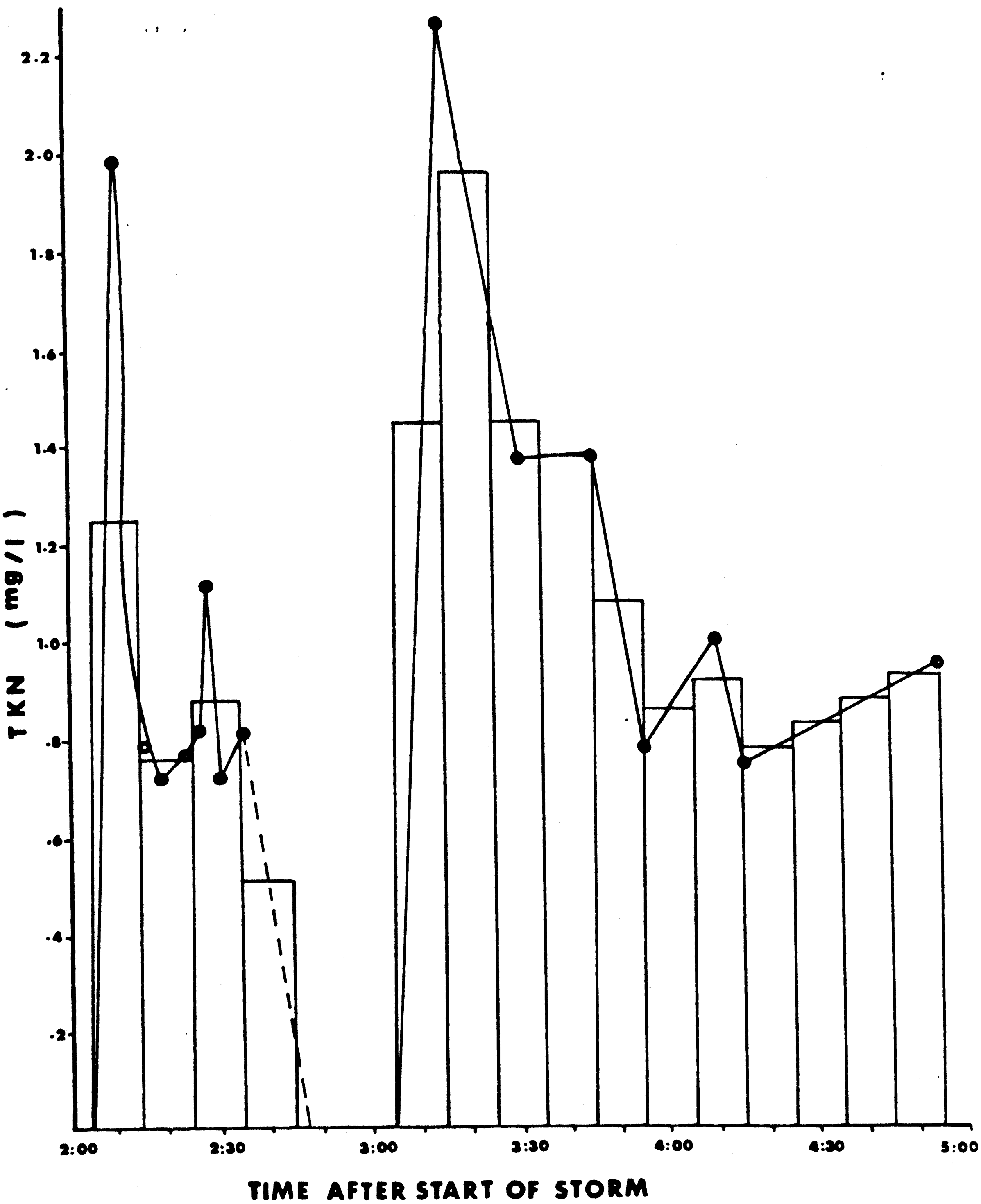

Fig. 17. Total Kjeldahl nitrogen pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 10 minute time increments for SWOPS time sensitivity analysis.

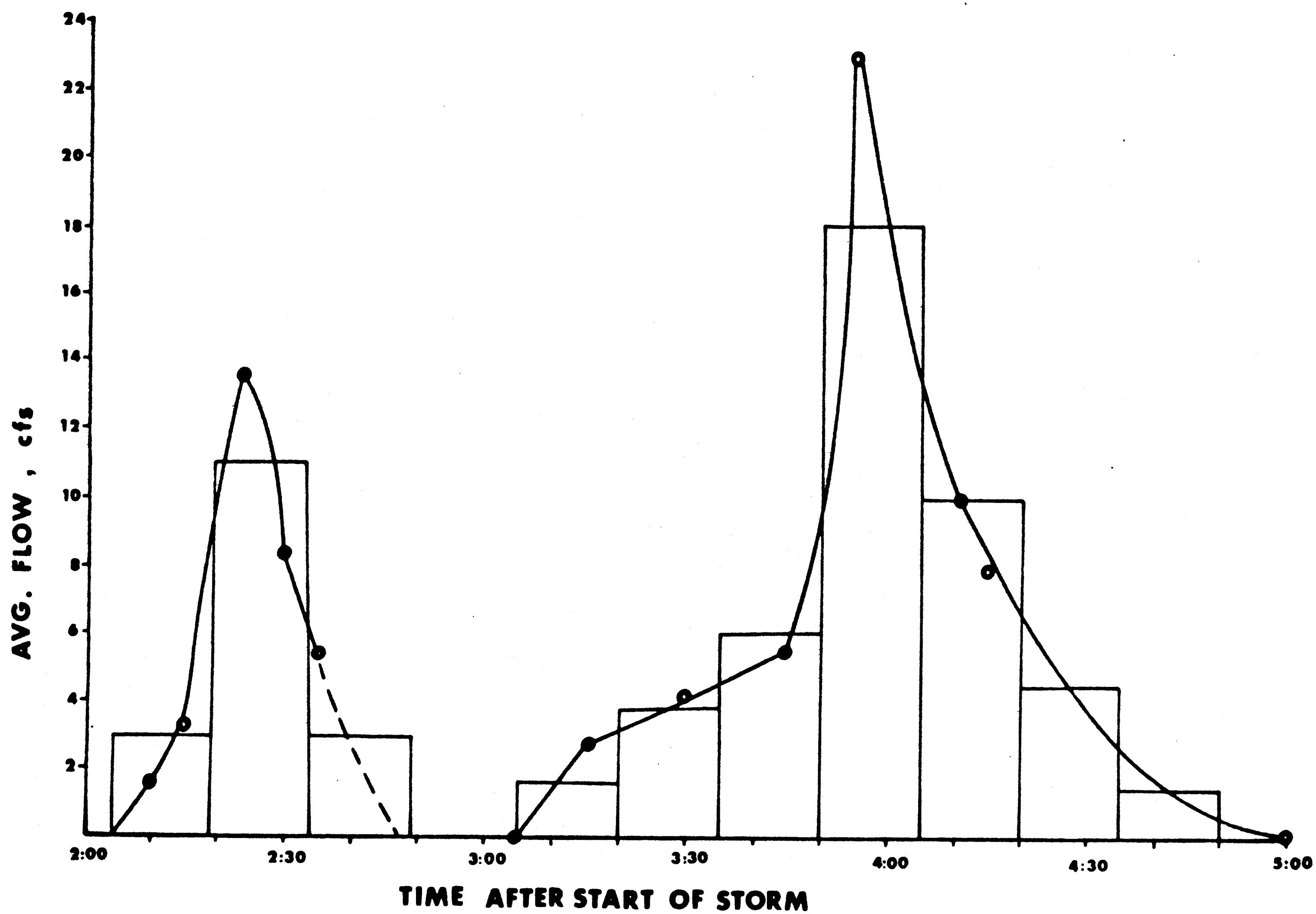

Fig. 18. Hydrographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments for SWOPS time sensitivity analysis.

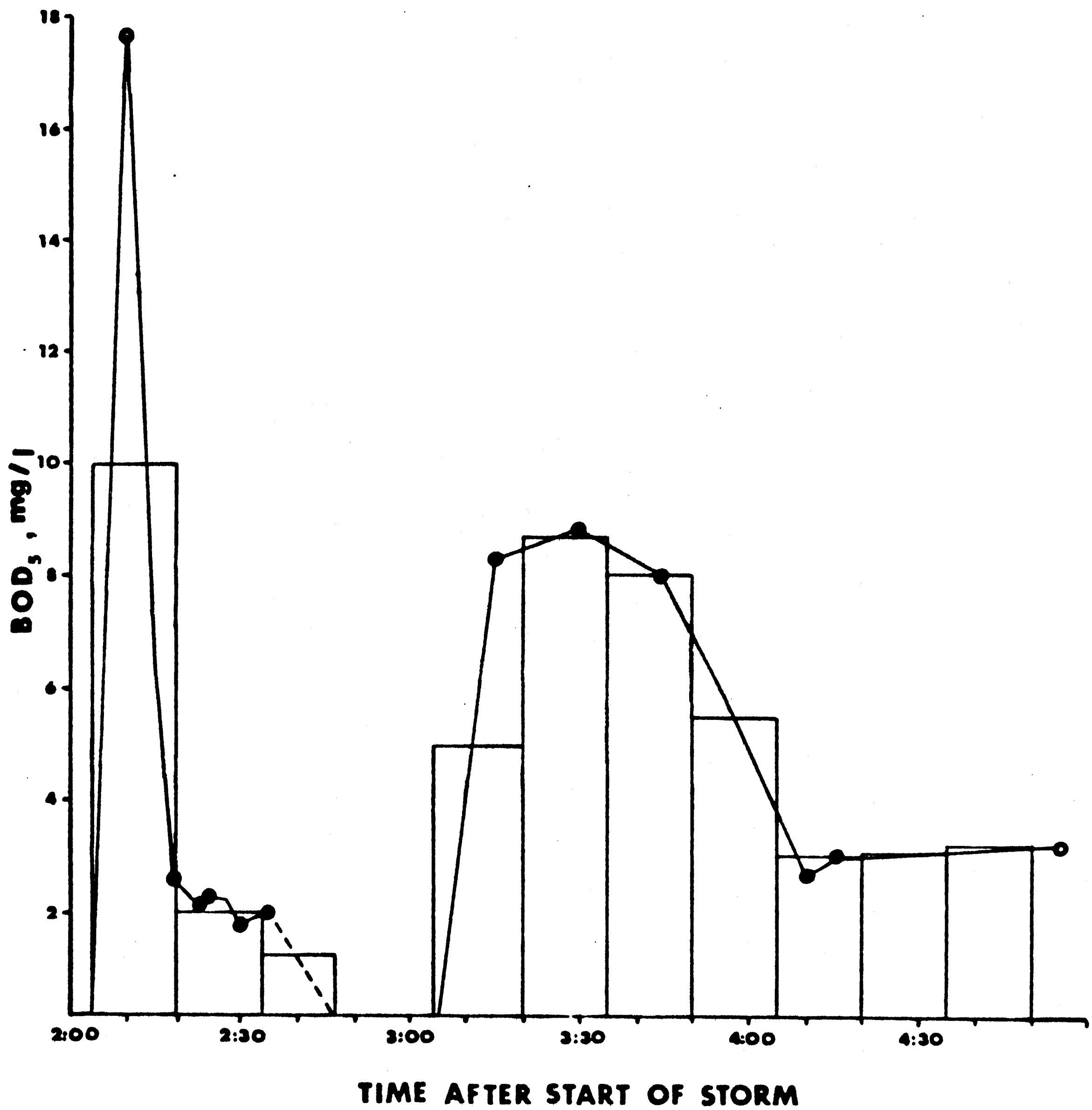

Fig. 19. Carbonaceous BOD_5 pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments for SWOPS time sensitivity analysis.

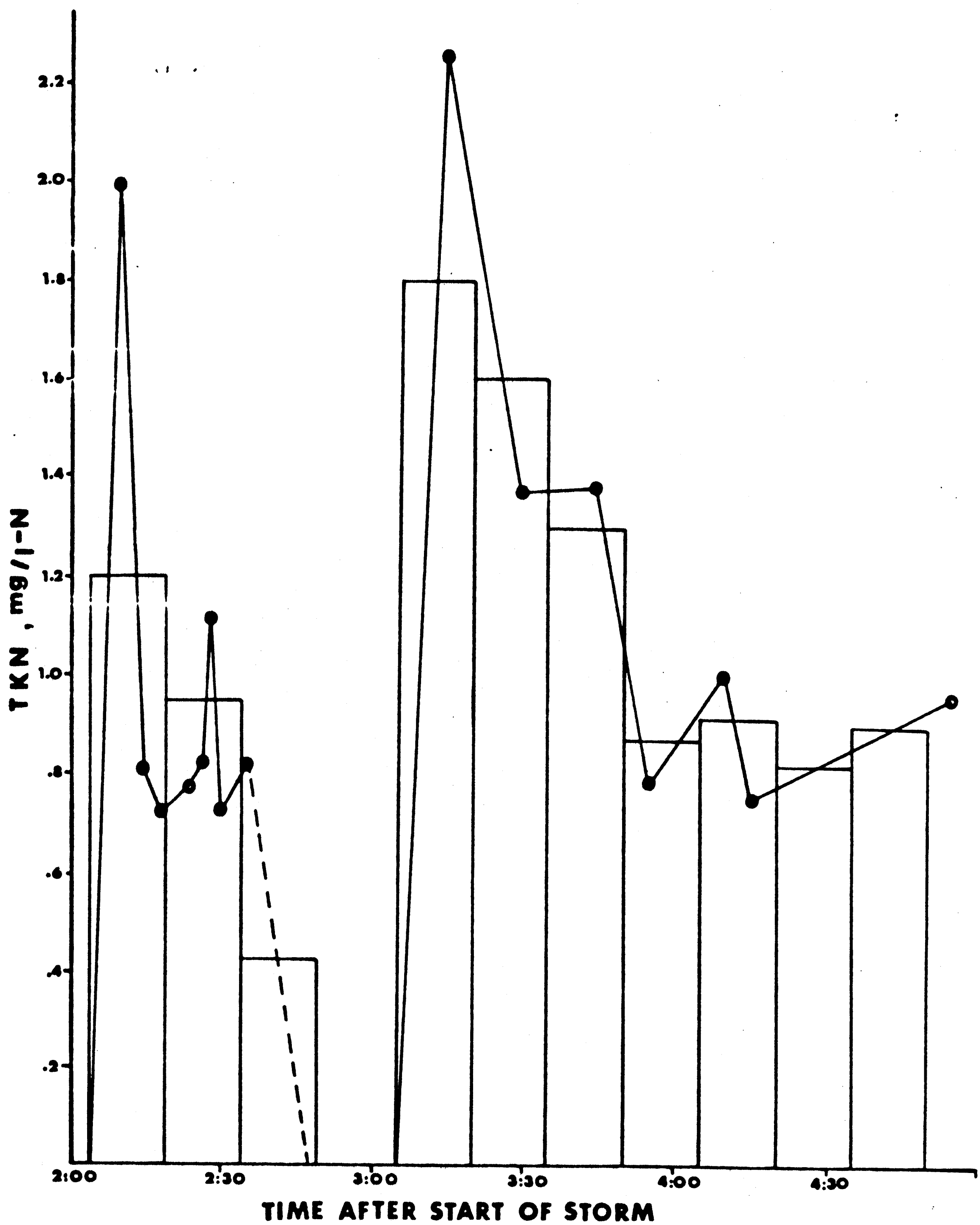

Fig. 20. Total Kjeldahl nitrogen pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments for SWOPS time sensitivity analysis.

TABLE 7

RESULTS OF TIME SENSITIVITY RUNS ON SWOPS

Storm Event	Size of Base Time Increment (# of Time Incr.)	Number of Time Increments Specified	% Accuracy to Base Time Increment Size	Second Moment / Time to Peak*First Moment
0.75 hour	5 min. (9)	3 (15 min.)*	69.9	0.199
		4 (10 min.)	78.0	
		5 (9 min.)	84.2	
		7 (6 min.)	91.0	
1.83 hour	5 min. (22)	5 (20 min.)	87.2	0.450
		7 (15 min.)	87.7	
		11 (10 min.)	98.5	
4.00 hour	10 min. (24)	8 (30 min.)	90.8	0.667
		12 (20 min.)	92.9	
		16 (15 min.)	96.3	
8.50 hour	20 min. (25)	5 (90 min.)	87.9	0.978
		8 (60 min.)	91.0	
		11 (45 min.)	91.8	
		17 (30 min.)	93.6	

* The time in parenthesis represents the size of time increment used.

The predicted peak BOD concentrations were observed to occur further downstream when larger time increments for hydrograph and pollutograph were used in SWOPS. This occurs since larger time increments correspond to larger distance increment calculations in the LaGrange Coordinate System. Therefore, the peak of the pollution pulse travels farther downstream. However, it was observed that the difference in downstream distance where the peak BOD concentration is predicted to occur, was less than 150 ft for any given time point. For the infinitely long stream simulated in SWOPS, this difference is insignificant. However, for much larger stream flows and time increment calculations, this difference in downstream distance would not be negligible and should be corrected by decreasing the size of the time increments specified.

In addition, it was also observed that the predicted BOD concentrations are initially significantly larger in the stream when larger time increments for hydrograph specification are specified. The larger the initial mass loading to the stream will be, resulting in the higher initial concentrations. This condition is quickly overcome after the first few time increments calculated since the peak pollutograph and hydrograph loadings are more accurately defined when smaller time increments are used. It was not unusual to observe a drop in BOD concentration variation from 50 percent to 5 percent within the first few time increments calculated in SWOPS. Once recovery has occurred, the concentrations predicted at smaller time increment specifications usually become larger than those predicted at the larger time increments. Again, this is due to the more accurate approximation of the rising limb, peak loading and receding limb of the storm events at smaller time increment analysis.

To develop a graphical relationship between the number of time increments specified and the dimensionless ratio (Equation 35) for the storm overflow events in Table 5, 90, 94 and 98 percent accuracies were chosen as desired targets. To obtain these accuracies, it was assumed that linear interpolation could be conducted between accuracies above and below 94 and 98 percent, therefore, giving the number of time increments subsequently required. The resulting time increment-shape curves are shown in Figure 21. The results of the linear interpolations for all the events are included, in addition to those accuracy points, contained in Table 7, which were required in order to carry out the procedure.

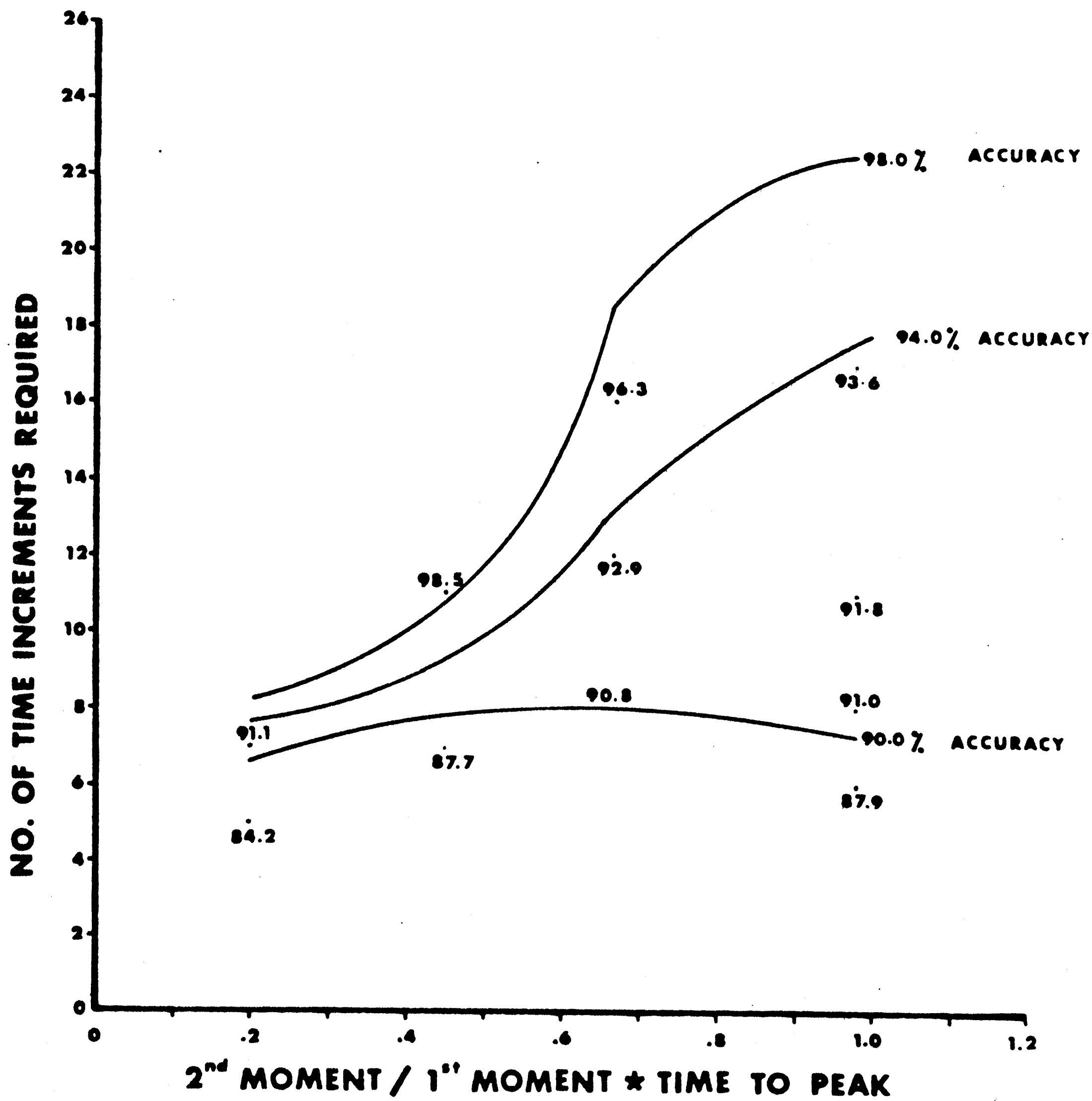

Fig. 21. Graphical illustration of the results obtained from the time sensitivity analysis conducted on SWOPS.

Curves relating SWOPS accuracy with the number of time increments specified revealed that the 90, 94 and 98 percent points obtained by linear interpolation for each event fell very close to their respective curves. These curves are not shown because more data points were required to show their complete shape. Time limitations prevented the determination of these points. Therefore, the assumption of linear interpolation to get a desired accuracy point is reasonable.

Figure 21 illustrates that the relationship between the dimensionless ratio and the number of time increments required to achieve desired accuracies with SWOPS is curvilinear. Essentially, as the dimensionless ratio, and hence the duration of the storm event, increases the number of time increments required must increase to achieve a required accuracy. The 90 percent accuracy curve followed this trend until a range of approximately 0.7 to 1.0 for the dimensionless ratio is reached. At this point, the number of time increments actually decreased with an increasing ratio. This suggests that the numerical integration scheme incorporated in SWOPS had difficulty converging on a solution for the large time increment sizes required in this range. As shown in Figure 21, the number of time increments required also increases for each event, as the desired accuracy increases. This results in the accuracy curves shifting upwards as the target accuracy increases.

The 94 and 98 percent accuracy curves reveal that there are ranges where an increase in the dimensionless shape ratio results in a deacceleration of the increase in the number of time increments required. For instance, to achieve 98 percent accuracy in SWOPS, the number of time increments required increased approximately from 8 to 18 as the dimensionless ratio increases from 0.2 to 0.65. However, the number of time increments required increased only from 18 to 22 for a ratio increase from 0.65 to 0.90. Since an increase in the size of the base time increment was required as the duration of the events increased (as explained earlier), the increase in the number of time increments required to achieve 98 percent accuracy was partially offset in this range for the dimensionless ratio. If the base time chosen was constant, the increase of the number of time increments required would have been substantial in this range.

Conversely, the large increase in the number of time increments required for a dimensionless ratio range of 0.2 to 0.65 is attributed to model convergence capabilities. The relatively short duration of the storms in this range results in SWOPS being highly sensitive to variations in the size and, hence, the number of time increments required to achieve the desired model accuracy. Smaller sizes of time increments are required, resulting in a larger increase in the number of increments required, from event to event, for SWOPS to converge at the desired accuracy.

To quantitatively define the relationship between the number of time increments required to obtain a desired accuracy and the dimensionless ratio for a particular stormwater or combined event, a logarithmic linear regression analysis was conducted for the 94 and 98 percent accuracy points obtained from linear interpolation of the data contained in Table 7. These points, as well as the resulting linear relationships, are shown in Figure 22.

The correlation coefficients obtained in the regression analysis for 94 and 98 percent accuracies were 0.915 and 0.925, respectively. This suggests that the variation in the interpolated points decreases as SWOPS accuracy increases. This is reasonable since the numerical integration technique can converge more readily on a particular point as the number of time increments describing a stormwater event(s) increases, therefore, increasing the accuracy of the model.

The slopes obtained for the linear log-log equation were 0.700 and 0.516 for 94 and 98 percent accuracies, respectively. This is a result of the larger increase in the number of time increments required to increase the model accuracy from 94 to 98 percent for an increasing dimensionless ratio (Figure 21). This is expected, since as storm duration increases, the variance (2nd moment) of the corresponding hydrograph increases, thereby requiring a greater number of time increments to increase the desired accuracy than would be required for storms of smaller duration.

Though the regression coefficients are not particularly good, it has been shown that there is a definite relationship between the number of time increments required and the dimensionless shape ratio, describing a stormwater or combined event, to arrive at a required SWOPS accuracy. Regions not investigated in this study, as shown by the dashed lines in Figure 22, should be analyzed in the future if this methodology is to be used in field applications. The additional points thus obtained should improve the correlation coefficients for both accuracies in Figure 22. If not, a different approach should be used to relate hydrograph shape and the number of time increments required. For instance, the dimensionless ratio 2nd moment/(1st moment)^2

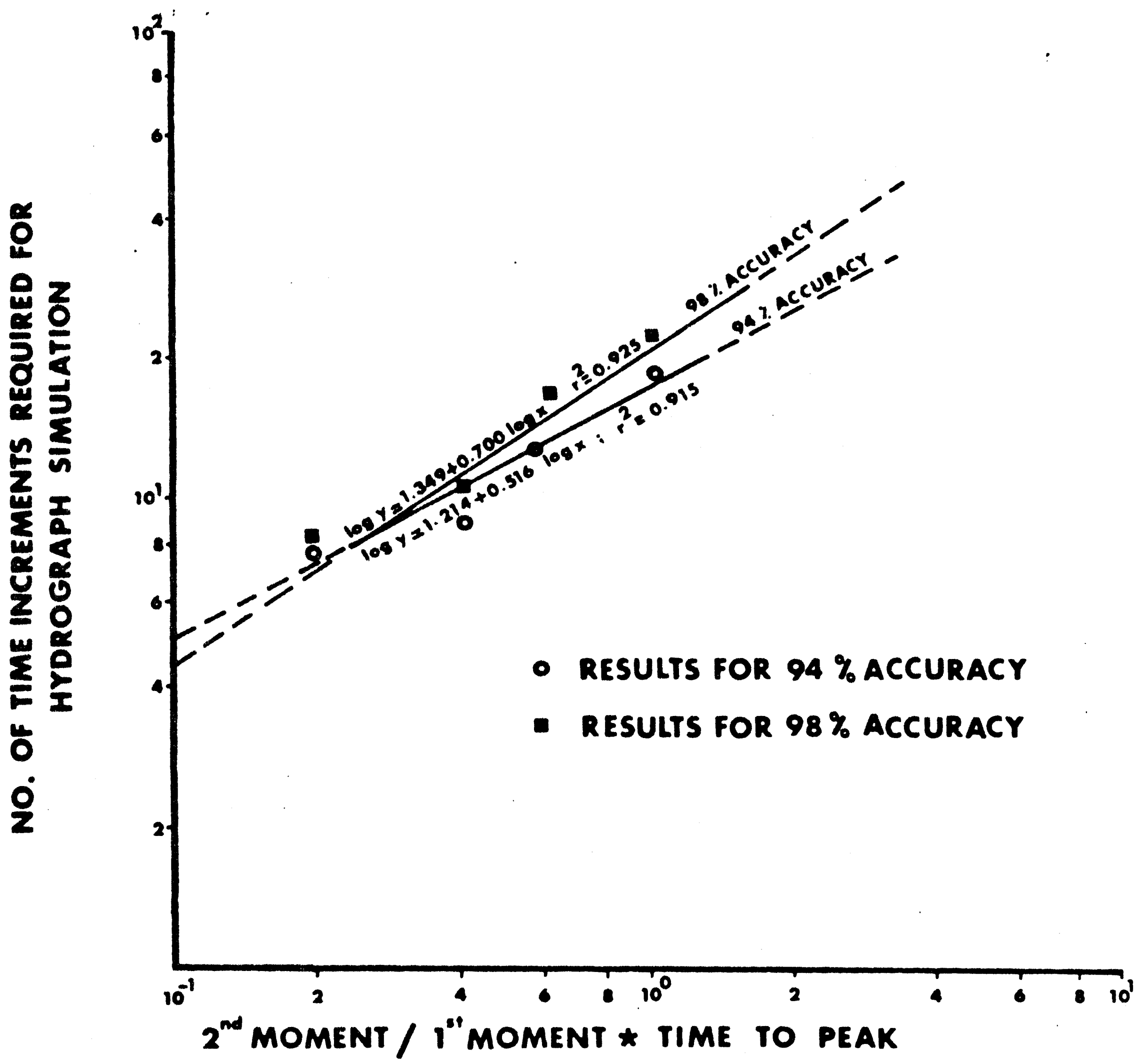

Fig. 22. Logarithmic interpretation of the results obtained from the time sensitivity analysis conducted on SWOPS.

might be used instead since these are the two most important parameters describing the shape of a statistical distribution. The third moment of the hydrograph, representing its skewness, might also be investigated for its impact on this relationship.

TRANSIENT MODELING OF MIXING ZONE REQUIREMENTS

SWOPS was utilized to investigate mixing zone requirements when a transient (non-point) loading is discharged into a hypothetical stream with constant kinetic, geometric and hydraulic characteristics. The objective is to develop a general approach to investigate the duration and magnitude of mixing zone standard violations. The approach can, therefore, serve as a guide to the engineer or planner in assessing if pretreatment (qualitative and quantitative) of a particular stormwater overflow event is required to meet current mixing zone requirements in an existing stream.

Since SWOPS is a one-dimensional model, mixing zone area (cross-sectional or surface) requirements cannot be evaluated without a high degree of uncertainty. Therefore, it is assumed that all stream responses, as measured in SWOPS, satisfy the 25 percent mixing zone cross-sectional area recommendation discussed earlier in the literature review chapter. This assumption is valid since the stream is infinitely wide as defined in SWOPS.

The hypothetical stream incorporated in this analysis is almost identical to the one used in the time sensitivity analysis, except that the reaeration coefficient, K_A, is reduced to 0.7 days^{-1}. This was necessary in order to insure measurable DO deficit responses for the stormwater overflow event utilized, therefore, allowing the average DO concentrations within the mixing zones to be calculated and compared to the standard.

The event used was the 8.5 hour storm shown in Figure 8, divided into 30 minute time increments. This event was chosen because it was the most potent event, in terms of quantity and quality parameters, of all those analyzed in the time sensitivity study. As stated previously, this event is representative of a combined sewer system overflow. Therefore, if mixing zone standards for the 8.5 hour storm are met, it can be deduced that these standards will also be met by utilizing the other stormwater events for identical stream conditions.

The mixing zone standards of interest are the 800 m length and the minimum average DO concentration requirement of 4.0 mg/l. The mixing zone length requirement is utilized due to the one-dimensional restriction imposed by using SWOPS. If any or both of these standards are violated, the stormwater overflow event produced unacceptable mixing zone conditions and pretreatment of the event would be required.

Mixing zone length and DO concentration requirements for the 8.5 hour storm were measured as a function of the initial DO concentration (DO_0) of the hypothetical stream. Since the initial DO concentration of the stream is not required as an input parameter in SWOPS, it can be arbitrarily chosen for the hypothetical stream examined. Site specific quantitative and qualitative data would be required for field applications of this analysis.

The two initial DO concentrations used in this analysis were 5.5 mg/l and 6.0 mg/l. SWOPS DO deficit concentration predictions were examined for 30 minute time increment calculations (8.5 hour storm) and the corresponding predicted DO concentration for a point downstream was calculated by subtracting the predicted DO deficit concentration from the assumed initial stream DO concentration. Mixing zone length requirements were analyzed by observing the downstream region where the predicted DO deficit concentrations would result in levels below the 5.0 mg/l standard required by the Florida Administrative Code, Chapter 17-3. The length was then calculated by subtracting the lower downstream distance point from the upper downstream distance point in this region for each time increment analyzed.

The mixing length requirements for both initial stream DO concentrations are shown in Figure 23. A semi-log graphical representation was necessary since the length requirements varied considerably, depending upon the initial DO_0 concentration chosen.

As can be seen in Figure 23, an initial stream DO concentration of 6.0 mg/l results in a maximum mixing length requirement of approximately 600 m, therefore, not exceeding the 800 m standard. This requirement occurred approximately 48.5 hours after the introduction of the 8.5 hour stormwater event in the stream. A lag phase of 12 hours was required before stream conditions deteriorated to such an extent that the predicted DO deficits fell below 1.0 mg/l. In comparison, only a 5.5 hour lag phase for DO_0 = 5.5 mg/l was required because of the lower initial stream quality assumed. In both cases the mixing length requirements steadily increased initially until decreasing biological oxidation, advective, dispersive and reaerative mechanisms allowed recovery to begin and finally improve stream conditions to such an extent that length requirements leveled off, and then

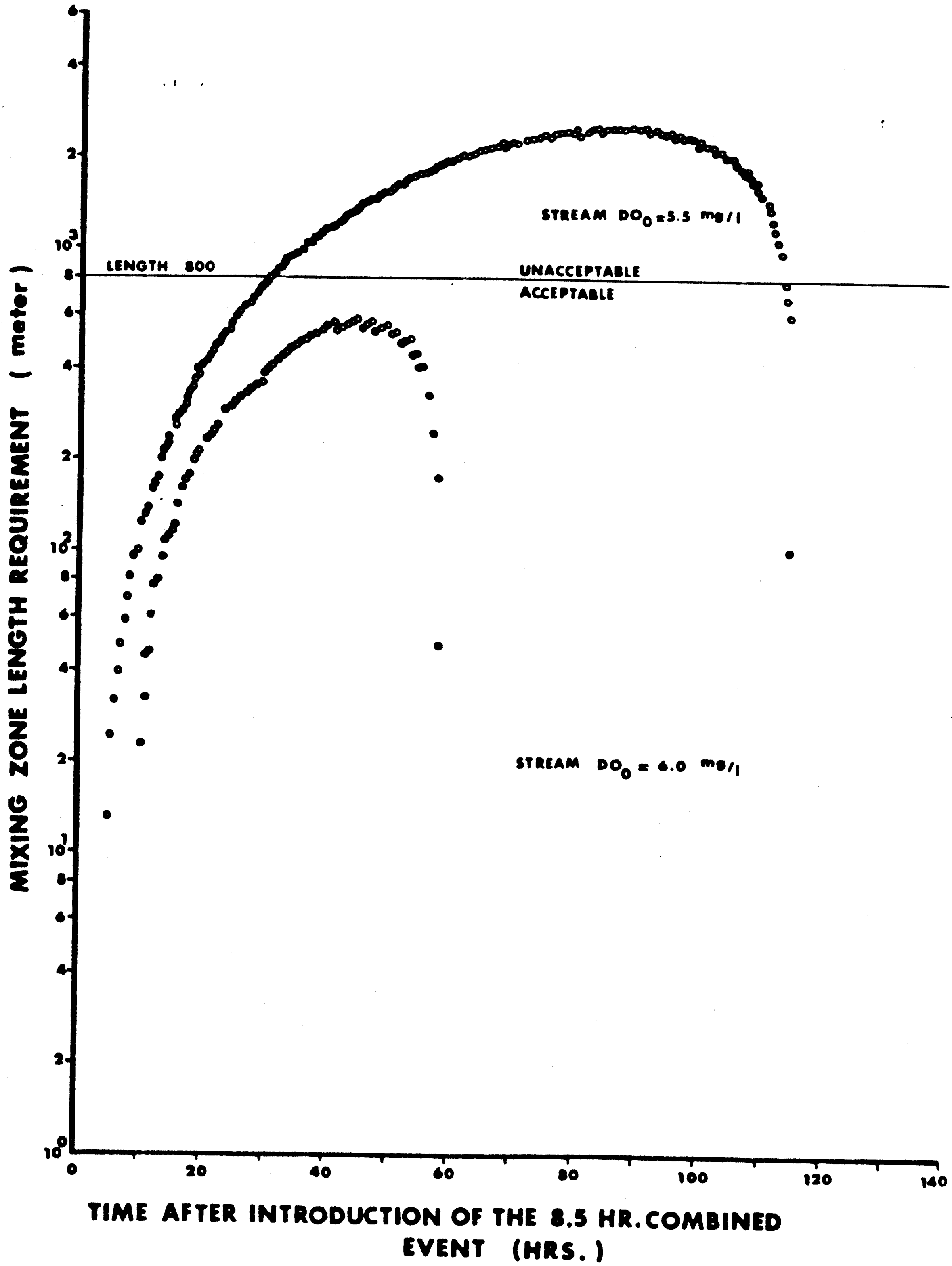

Fig. 23. Mixing zone length curves derived for a hypothetical stream by using SWOPS.

decrease sharply until no mixing zone was required. This phenomenon is of longer duration for lower initial stream water quality (110 hours for D_0 = 5.5 mg/l) than for higher quality receiving water (49 hours for D_0 = 6.0 mg/l).

The 800 m mixing zone length standard was exceeded for 85 hours when the initial stream DO concentration was assumed to be equal to 5.5 mg/l, culminating at a mixing length of approximately 2564 m at 88.5 hours after the introduction of the storm overflow event. Therefore, the stream mixing zone length requirements are unacceptable from hours 30.5 to 113.5 after discharge. If this was an existing field condition, the planner would want to examine the economic and technical feasibility of incorporating pretreatment control strategies to eliminate this problem. Stormwater retention systems could be utilized to quantitatively and qualitatively treat the runoff from the first inch of rainfall.

The ripples observed in the mixing length curves contained in Figure 23 are a result of the iterative numerical integration technique incorporated in SWOPS. Basically, for each time incremented calculated, a DO or BOD concentration is predicted for a distance increment in the stream reach. A distance increment is increased by a distance unit, DX (see Table 2) for each time increment increased by DT (DX = DT * VEL). Therefore, as the number of time increments increase so will the number of distance increments needed to specify a mixing zone length if stream water quality is deteriorating. As can be seen in Figure 23, the ascending limb in the curves are made up of small groups of computed mixing zone length points. When an additional distance increment is required to include the spread of pollutants downstream, the points jump accordingly. As the mixing zone length requirements stabilize, this effect is less pronounced, therefore, forming the associated hump on the curve. When stream recovery begins to occur (receding limb) the decrease in distance increment requirements result in the opposite. The group of points descend accordingly. This can be corrected by specifying a smaller DT, thereby effectively reducing DX and providing a more consistent distance field.

Figure 24 illustrates the DO sag curves experienced within the mixing zones. The results presented are average concentrations. As expected, decreasing the initial stream DO concentration produces a wider and lower sag curve. The minimum average DO concentrations for DO_0 equal to 5.5 and

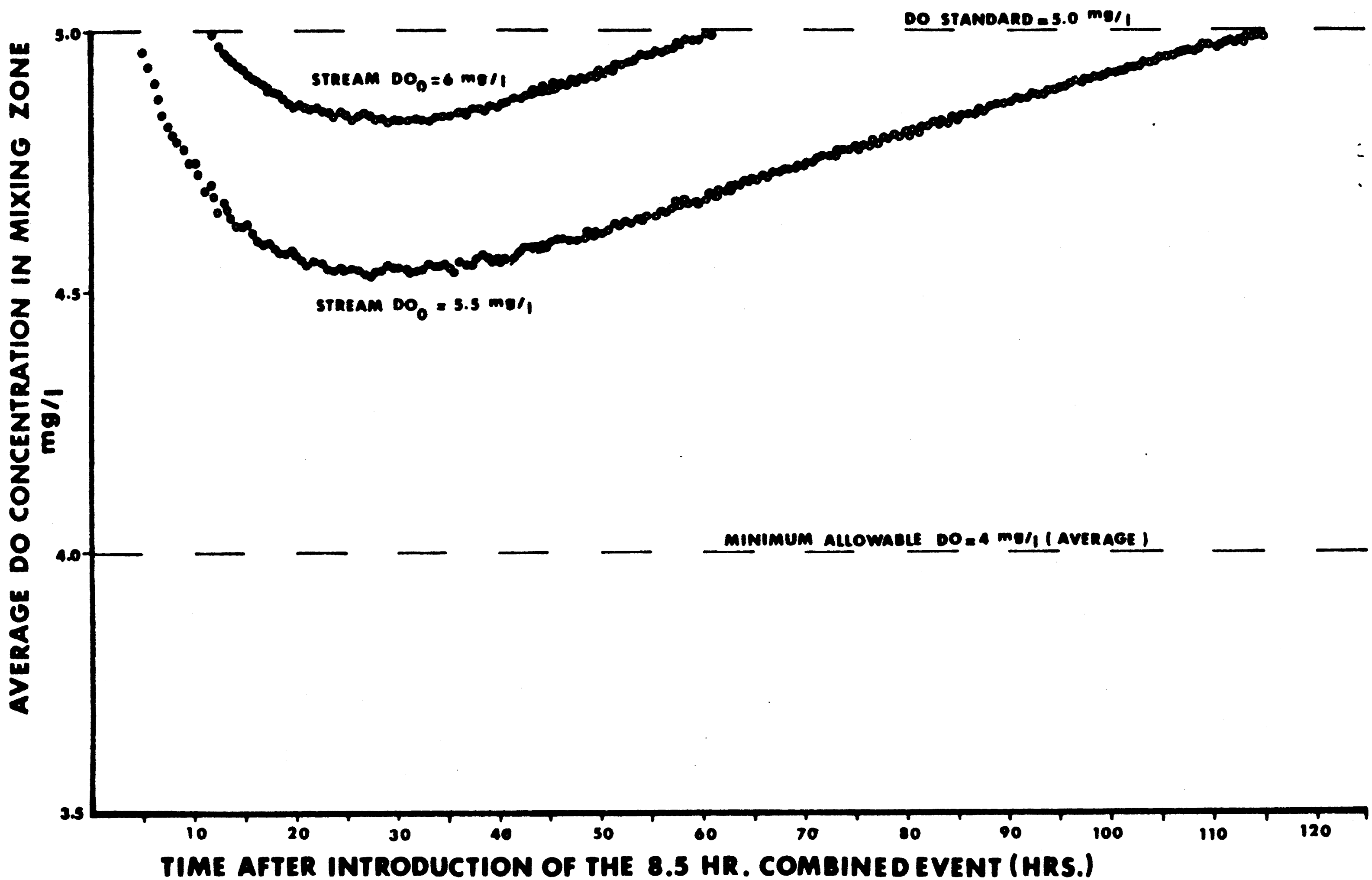

Fig. 24. Oxygen sag curve derived from the mixing zone length analysis using SWOPS.

6.0 mg/l are approximately 4.53 and 4.83 mg/l, respectively. These concentrations occur at 27, 27.5 and 31.0 hours for D_0 = 5.5 mg/l and at approximately 27.5-32.5 hours for D_0 = 6.0 mg/l after introduction of the stormwater overflow event. The curves do not violate the minimum allowable average DO concentration of 4.0 mg/l in mixing zones. The duration of the sag curves are equivalent to the corresponding durations of the mixing length curves shown in Figure 23. Essentially, as the number of distance increments required to define the mixing length increase, the average DO concentration will decrease. The opposite will result when the number of distance increments required decreases.

To compare the transient mixing zone length requirements for a stormwater overflow event with that obtained for the 8.5 hour combined event, the 1.83 hour storm shown in Figures 12-14 was analyzed for the same quantitative and qualitative stream conditions. The 1.83 hour storm was divided into 5 minute increments.

The results revealed that no mixing zone was required for the 1.83 hour storm for both initial DO concentrations in the stream examined earliers. The dissolved oxygen deficits predicted by SWOPS were minimal, never being greater than 0.09 mg/l for any time period under investigation.

Since the 0.75 and 1.83 hour storms were observed in the field, it may be that stormwater events have minimal, if any, impact on mixing zone requirements in streams or rivers. The 0.75 hour storm is "weaker" in strength than the 1.83 hour storm; therefore, it would have a lesser impact on the stream than the latter event. The conclusion is justifiable since the stream reach modeled in this study had a flowrate of only 5 cfs, a flowrate which is very small when compared to those encountered in existing streams.

The results suggest that non-point discharges from combined outfall systems can lead to serious violations of existing mixing zone standards depending on the qualitative characteristics of the stream in question. Pretreatment control strategies would then be required to insure that mixing zone standards are adhered to.

It should be realized that errors are introduced in the numerical integration technique utilized in SWOPS. Unfortunately, the errors in predicting BOD or DO concentrations have never been quantified against field data. Comparison of SWOPS with the Streeter-Phelps Model has produced very good results (EPA 1978). Therefore, the predicted concentrations in SWOPS should not be misrepresented as actual concentrations but should be presented as approximated results. Standard DO probes produce an accuracy of ±0.1 mg/l from actual field concentrations (American Public Health Association 1981), hence, no DO measurement techniques are 100 percent accurate. SWOPS should give results with an equal or greater degree of accuracy.

The long duration period required for DO_0 = 5.5 mg/l in Figure 23 is a result of the values assumed for the biological, kinetic, dispersive and hydraulic parameters in the hypothetical stream. Therefore, the "spread" and "peak" of this curve can be effectively reduced if the stream in question had a higher flowrate, therefore, providing a greater dilution potential to exist. Increasing the dispersion coefficients for BOD and DO would increase the longitudinal spread of pollutants due to turbulent diffusion. Increasing the reaeration coefficient would resupply the DO levels in the stream due to surface diffusion. Each of these parameters would effectively increase the predicted concentrations of DO, therefore, decreasing the length and DO requirements for resulting mixing zones. Increasing the rate constants for BOD removal due to deoxygenation and combined deoxygenation and sedimentation would result in lower predicted DO concentrations, therefore, effectively increasing and decreasing mixing zone length and average DO concentrations, respectively.

CHAPTER VII

MIXING ZONE MODEL FOR LAKES

A finite element computational procedure was developed into a mathematical model called LAKE. The finite element procedure has obtained popularity for the modeling of fluid systems (S. Wang, 1980). Others also have examined the use of finite elements in lake modeling, but some were only used for steady state modeling (Pinder and Gray, 1977). Others were under development at the time of this research but not well documented (J. Wang, 1981).

In the LAKE model, the governing partial differential equations express the law of conservation-of-mass for the pollutant and for dissolved oxygen. The Galerkin finite element method is used to numerically integrate the governing equations in the space dimensions. An implicit method of numerical integration is used in the time dimension. The LAKE model is similar to DISPER except that numerical integration in the time domain is performed implicitly in LAKE and explicitly in DISPER (Leimkuhler, 1975).

MODEL DEVELOPMENT

Using an X-Y coordinate system, let the coordinates of any node on the lake surface be specified as (x,y). The vertical component of fluid velocity is assumed zero and fluid velocities in the horizontal plane are v(x,y) in the X direction and w(x,y) in the Y direction. The depth of the lake at each node is h(x,y) and the depth for any element is the mean of the depths for the bounding nodes. The depth averaged concentration of the pollutant at any node is C(x,y) in gm/m^3 and the depth averaged concentration of dissolved oxygen deficit is D(x,y) also as gm/m^3.

Dissolved oxygen deficit is defined as the saturation value of dissolved oxygen at the water temperature minus the dissolved oxygen concentration. Consider a column of water within the lake. The time rate at which the pollutant accumulates within the differential water column is the sum of the advective mass flux, the dispersive mass flux, the pollutant point sources and the rate of biological degradation within the column. The rate of accumulation of dissolved oxygen deficit is the sum of the advective mass flux, the dispersive mass flux, oxygen mass transfer rate from the atmosphere and the rate of biological usage of dissolved oxygen. Advective flux of the pollutant entering the column in the X and Y directions is Cvhdy and Cwhdx. Advective flux leaving the column in the X and Y directions is Cvhdy + $(Cvh)_x$dxdy and Cwhdx + $(Cwh)_y$dydx, where the subscripts indicate partial differentiation. Since mass accumulated must equal mass flux in minus mass flux out, the governing differential equation when only advective mass transfer is considered is written as: $hC_t = -(Cvh)_x - (Cwh)_y$.

The rate of pollutant accumulation within the column resulting from dispersive flux can be computed in a similar way. Let E (m^2/sec) be the dispersion coefficient in both the X and Y directions. The dispersive mass flux entering the column in the X direction is $-EC_x hdy$. Since the mass flux leaving the column in the X direction is $-E(C_x h + (C_x h)_x dx)dy$, the time rate of accumulation is the difference or $E(C_x h)_x dxdy$. Similarly, the time rate of accumulation in the Y direction is expressed as $E(C_y h)_y dxdy$. Therefore, if only dispersive accumulation of the contaminant is considered, the governing equation is written as: $hC_t = E((C_x h)_x + (C_y h)_y)$.

Since the LAKE model can also be used for analyzing the pollutional impact of stormwater overflows on streams, dispersion coefficients along (E_{long}) and normal to (E_{lat}) the stream velocity vector have been used in the LAKE program. If Θ is the angle between the stream velocity vector and the position X axis, three new dispersion coefficients are computed from E_{long}, E_{lat} and Θ as follows:

$$E_{xx} = E_{long}\cos^2\Theta + E_{lat}\sin^2\Theta$$

$$E_{yy} = E_{long}\sin^2\Theta + E_{lat}\cos^2\Theta$$

$$E_{xy} = (E_{long} - E_{lat})\sin\Theta\cos\Theta$$

Using these new dispersion coefficients, the governing equation for dispersive flux only is written as follows: $hC_t = E_{xx}(C_x h)_x + E_{xy}[(C_x h)_y + (C_y h)_x) + E_{yy}(C_y h)_y]$. When E_{long} equals E_{lat} both can be written as E and the governing equation for dispersion only reduces to: $hC_t = ((C_x h)_x + (C_y h)_y)E$.

If the time rate of pollutant degradation is taken as a fraction of the pollutant concentration per unit of time, K_r (sec^{-1}) the mass balance relationship for degradation alone is written as: $C_t = -K_r C$.

The computational program allows for specification of point sources of pollutant at any of the nodal points. Input data include the node number, the starting time (sec), the ending time (sec) and a vector of up to ten numerical source values (gm/sec) spaced at the time points separated by the delta time (DT) in seconds. If the symbol P (gm/sec) is used to indicate the magnitude of the point source, the governing equation has the form: $C_t dxdyh = P$ or $hC_t = P/dA$ where dA is the incremental surface area. The complete governing equation for the biodegradable pollutant can be written as follows, where the concentration is averaged over depth.

$$hC_t = -(Cvh)_x - (Cwh)_y + E_{xx}(C_x h)_x + E_{xy}((C_x h)_y + (C_y h)_x) - K_r hC + P/dA \quad (15)$$

The governing equation for dissolved oxygen concentration (gm/m^3) in the LAKE model is more conveniently expressed in terms of dissolved oxygen deficit, D (gm/m^3). If K_d (sec^{-1}) is taken as the deoxygenation rate related to the rate of pollutant degradation K_r, and K_L (m/sec) is taken as the oxygen transfer coefficient, the governing equation for dissolved oxygen deficit in the differential water column can be written as follows:

$$hD_t = -(Dvh)_x - (Dwh)_y + E_{xx}(D_x h)_x + E_{xy}((D_x h)_y + (D_y h)_x) + E_{yy}(D_y h)_y + K_d hD - K_L hD \quad (16)$$

The rate at which oxygen from the atmosphere diffuses across the lake surface film is usually expressed[3] as follows:

$$q = AE_s(D/f)$$

q = rate of oxygen transport through lake surface, gm/sec

E_s = molecular diffusivity of oxygen in water, m^2/sec

A = lake surface area, m^2

D = dissolved oxygen deficit in lake water, gm/m^3

f = film thickness, m

The grouping (E_s/f) is commonly referred to as K_L, the oxygen transfer coefficient. As shown in Leimkuhler (1975), a number of empirically derived relationships are available for estimation of K_L. For example, one such relationship is: $K_L = 1.57 + 0.32V_w^2$ where the dimensions of K_L are ft/day and dimensions for wind velocity (V_w) are ft/sec. Another set of empirical relationships is: $K_L = 0.362V_w^{\frac{1}{2}}$ for V_w less than 5.5 m/sec and $K_L = 0.0277V_w^2$ for V_w greater than 5.5 m/sec. Dimensions for K_L in these equations are m/day. Integration of the Governing Equation will be shown in Appendix C.

CHAPTER VIII

LAKE MODEL EXECUTIONS

HOW TO USE THE PROGRAM

The first step is to divide the surface of the lake or stream into any number of triangular elements. The greater the number of triangles, the greater the accuracy. Extreme points of the triangles will be nodes. Lay out an X-Y coordinate system and measure the X and Y coordinates of each node. Fine the water depth at each node. Count the number of nodes and the number of elements. Read in the number of nodes as NNOD and the number of elements at NELM. Determine which of the nodes are point sources. Find initial values for C and D at each of the nodal points. Estimate fluid velocity X and Y components at each node (UX and UY). For each node read in C, D, P, X, Y, H, UX and UY (see Appendix for input definitions). P will have a value of 1 for active nodes and a value of 0 for inactive nodes. Read in values for ELONG, ELAT, DT, ME, NOPS, KR, KD, KL and ANGLE (see Appendix A for input definitions). Make a table with one row for each element and three columns. In the row for the I'th element enter the number of the three bounding nodes in counterclockwise order. The starting node number is not important. Read this table in as NOD(I,J). Next, divide the pollutograph for each active node into ten time increments each DT in length. Find the pollutant discharge in gm/sec in each time increment. Read this information in as PDATA where the first element is the node number, the second is the start time (sec), the third element is the stop time, and the next ten elements are the rates of discharge (gm/sec). If the pollutant source is continuous make the stop time greater than MTDT and put the rate in the fourth position of PDATA.

As an example, consider the channel shown in Figure 25. This example simulates a portion of a very wide channel with point sources spaced at equal intervals along a line to represent a true line source of pollutants, possibly stormwater. This example is used because a closed form (analytic) solution can be found by superposition.

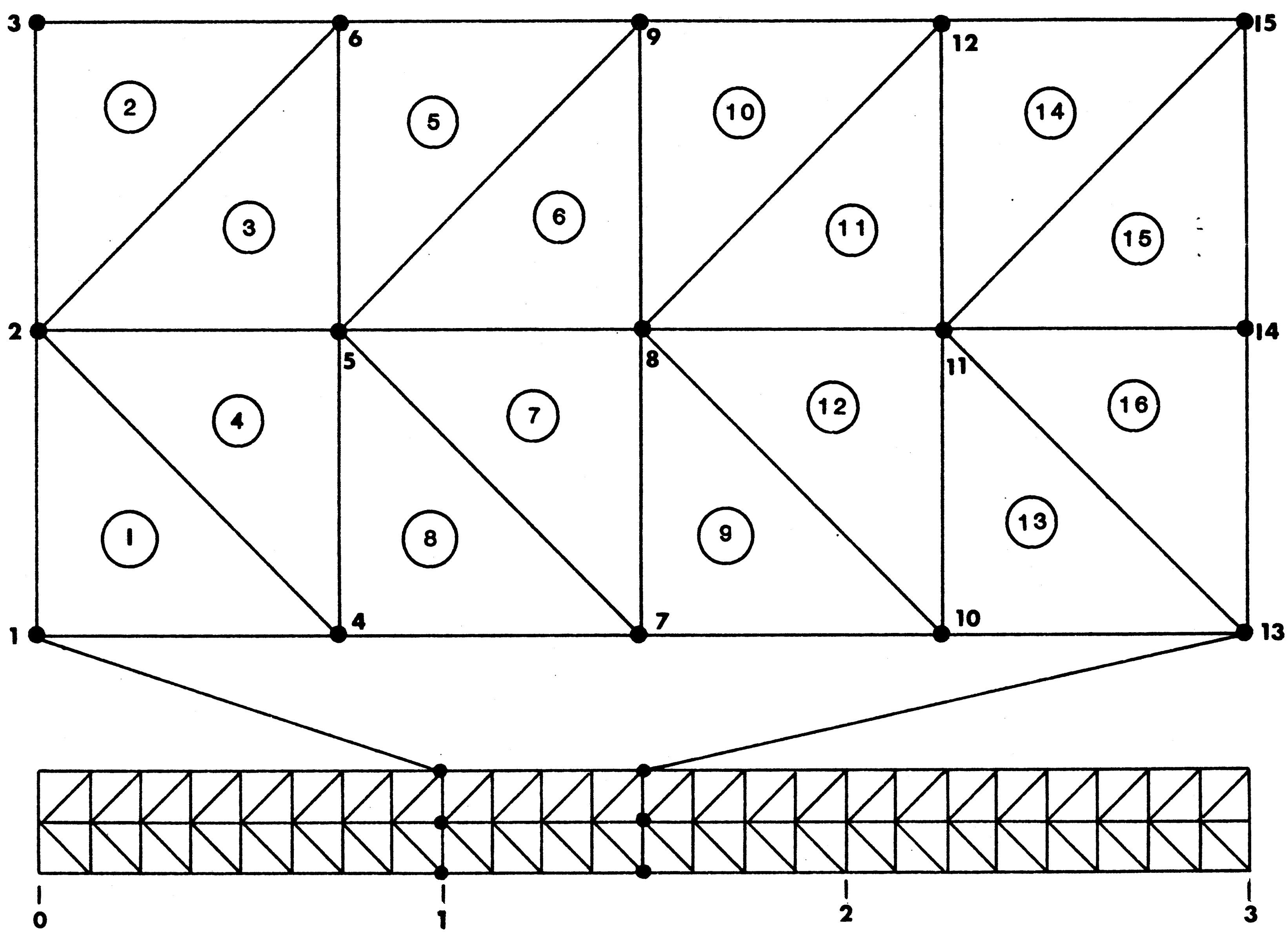

Fig. **25**: Discretization scheme used for channel dispersion to compare analytical and numerical solutions.

The spacing of the square grid is 0.125 meters so X(5) = .125 and Y(5) = .125. The depth of all nodes is taken as 1 meter. Both ELONG and ELAT are set equal to 0.01 m^2/sec. K_r, K_d and K_L are all zero. The time increment (DT) is taken as 0.1 sec and the number of time increments (MT) is 30. The matrix (NOD) has the following form.

ELEMENT NUMBER	NODE NUMBERS (COUNTERCLOCKWISE DIRECTION)		
1	1	4	2
2	2	6	3
3	2	5	6
4	4	5	2
5	5	9	6
6	5	8	9
7	5	7	8
8	4	7	5
9	7	10	8
10	8	12	9
11	8	11	12
12	10	11	8
13	10	13	11
14	11	15	12
15	11	14	15
16	13	14	11

The three point sources are taken as continuous with a rate for node 1 of .125 gm/sec, a rate for node 2 of 0.25 gm/sec and a rate for node 3 of .125 gm/sec. Therefore, the three PDATA vectors are written as follows.

1	0	4	.125	0	0	0	0	0	0	0	0	0
2	0	4	.25	0	0	0	0	0	0	0	0	0
3	0	4	.125	0	0	0	0	0	0	0	0	0

Physically, the phenomenon being simulated is a line source of pollutant in water which covers an infinite plane to a depth of one meter. The magnitude of the line source is 4 grams/sec per meter of length. This is approximated by having the point sources of 0.5 gm/sec spaced at 0.125 meters along the line source. The diagram shown in Figure 25 is a segment of the infinite plane bounded by planes along the boundaries perpendicular to the plane covered with water. These bounding planes divide the point sources. Thus, node 2 is halved to 0.25 gm/sec and nodes 1 and 3 are halved twice giving a source strength of 0.125 gm/sec.

For the sample problem shown in Figure 25; the A, F and E matrices are all banded with a band width (NBAND) of 9. Thus, computer storage space is minimized by transforming the 15x15 matrices into matrices with 15 rows and 9 columns. The transformation is performed by placing the element in the (I,J) position of the 15x15 matrices in the position (I, MS+j-I) in the 15x9 matrices. The number MS is computed as (NBAND+1)/2 or t in the sample problem. AFter the transformation elements on the diagonal of the square matrices will appear in the MS'th column of the transformed matrices. In the sample problem all elements of the F matrix are null because the velocity components were taken as zero. The transformed A matrix with all elements multiplied by 1000 is shown below.

0.0	0.0	0.0	0.0	1.302	0.651	0.0	0.651	0.0
0.0	0.0	0.0	0.651	5.208	0.651	1.302	1.302	1.302
0.0	0.0	0.0	0.651	1.302	0.0	0.0	0.651	0.0
0.0	0.651	1.302	0.0	3.906	1.302	0.0	0.651	0.0
0.0	1.302	0.0	1.302	7.812	1.302	1.302	1.302	1.302
1.302	0.651	0.0	1.302	3.906	0.0	0.0	0.651	0.0
0.0	0.651	1.302	0.0	3.906	1.302	0.0	0.651	0.0
0.0	1.302	0.0	1.302	7.812	1.302	1.302	1.302	1.302
1.302	0.651	0.0	1.302	3.906	0.0	0.0	0.651	0.0
0.0	0.651	1.302	0.0	3.906	1.302	0.0	0.651	0.0
0.0	1.302	0.0	1.302	7.812	1.302	1.302	1.302	1.302
1.302	0.651	0.0	1.302	3.906	0.0	0.0	0.651	0.0
0.0	0.651	1.302	0.0	2.604	0.651	0.0	0.0	0.0
0.0	1.302	0.0	0.651	2.604	0.651	0.0	0.0	0.0
1.302	0.651	0.0	0.651	2.604	0.0	0.0	0.0	0.0

Similarly, the transformed E matrix with all elements multiplied by 1000 is shown below.

0.0	0.0	0.0	0.0	10.0	-5.0	0.0	-5.0	0.0
0.0	0.0	0.0	-5.0	20.0	-5.0	0.0	-10.0	0.0
0.0	0.0	0.0	-5.0	10.0	0.0	0.0	-5.0	0.0
0.0	-5.0	0.0	0.0	20.0	-10.0	0.0	-5.0	0.0
0.0	-10.0	0.0	-10.0	40.0	-10.0	0.0	-10.0	0.0
0.0	-5.0	0.0	-10.0	20.0	0.0	0.0	-5.0	0.0
0.0	-5.0	0.0	0.0	20.0	-10.0	0.0	-5.0	0.0
0.0	-10.0	0.0	-10.0	40.0	-10.0	0.0	-10.0	0.0
0.0	-5.0	0.0	-10.0	20.0	0.0	0.0	-5.0	0.0
0.0	-5.0	0.0	0.0	20.0	-10.0	0.0	-5.0	0.0
0.0	-10.0	0.0	-10.0	40.0	-10.0	0.0	-10.0	0.0
0.0	-5.0	0.0	-10.0	20.0	0.0	0.0	-5.0	0.0
0.0	-5.0	0.0	0.0	10.0	-5.0	0.0	0.0	0.0
0.0	-10.0	0.0	-5.0	20.0	-5.0	0.0	0.0	0.0
0.0	-5.0	0.0	-5.0	10.0	0.0	0.0	0.0	0.0

COMPARISON OF LAKE RESULTS WITH ANALYTIC SOLUTION

An analytical solution for the concentration at any point (node) in an infinite plane as a function of time resulting from an impulsive input of pollutant is available from the literature (Zison). If (d) is the distance in meters from the impulsive point source to the node and (t) is the time from occurrence of the impulsive input in seconds; the concentration C(d,t) in grams/m^3 can be computed using the following relationship when the water depth is one meter everywhere in the infinite plane.

$$C(d,t) = (0.0796M/(Et))/\exp(d^2/4Et))$$

In this equation E is the dispersion coefficient in m^2/sec assumed to be independent of direction. If the point source of pollutant is continuous with a magnitude of P (gm/sec) it can be approximated as a series of impulsive loads of magnitude PDT where DT is the time between impulsive loads. Since the differential equations describing dispersion are linear, the superposition principle applies and the concentration at any node can be computed as the summation of the effects of many impulsive loads spaced at intervals of DT in time. A digital computer program (ALAKE) has been developed to carry out this computation.

The LAKE program was used to compute concentration as a function of time at five nodes spaced at intervals of 0.125 meters along a line perpendicular to the line source. The line source was simulated by continuous sources of magnitude 0.5 gm/sec and the continuous sources were simulated by impulsive loads spaced at intervals of 0.1 sec. Seven stormwater inputs were used, one at the first node and three above and three below the line of nodes. The spacing for the point sources was 0.125 meters. Thus, this simulation should give the same results as the sample problem shown in Figure 25 which was computed using the LAKE finite element procedure.

A comparison of results computed with LAKE and the analytical solution for the infinite plane problem are shown in Table 8. The agreement is not strikingly good, but the two sets of numbers are very similar. Leimkuhler (1975) empirically determined a rule of thumb for the DISPER program which states that the incremental time (DT) must be less than the characteristic length squared divided by ten times the dispersion coefficient. For the infinite plane problem, the characteristic length, (0.125 m) squared, divided by the dispersion coefficient, (0.01 m^2/sec), divided by 10 equals 0.156 sec as the allowable DT. Since a time increment of 0.1 seconds was used, the accuracy of the finite element simulation would have been judged adequate using this rule of thumb.

TABLE 8: Comparison of concentrations computed with LAKE and an analytical solution for a line source of pollutant in an infinite plane. Numbers in parentheses are computed with LAKE.

	Node Numbers				
Time, sec	2	5	8	11	14
0.3	13.28 (9.82)	0.7 (0.08)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
0.6	18.39 (15.57)	2.78 (2.05)	0.17 (0.0)	0.0 (0.0)	0.0 (0.0)
0.9	22.32 (19.80)	5.10 (4.41)	0.65 (0.29)	0.04 (0.0)	0.0 (0.0)
1.2	25.62 (23.29)	7.37 (6.73)	1.35 (0.98)	0.16 (0.0)	0.01 (0.0)
1.5	28.52 (26.34)	9.54 (8.95)	2.30 (1.89)	0.37 (0.15)	0.04 (0.0)
1.8	31.12 (29.08)	11.59 (11.07)	3.31 (2.91)	0.68 (0.41)	0.10 (0.01)
2.1	33.49 (31.85)	13.52 (13.08)	4.37 (4.0)	1.07 (0.78)	0.19 (0.15)
2.4	35.67 (33.90)	15.35 (15.0)	5.45 (5.13)	1.52 (1.24)	0.33 (0.39)
2.7	37.68 (36.08)	17.08 (16.84)	6.54 (6.29)	2.03 (1.79)	0.50 (0.73)
3.0	39.56 (38.13)	18.72 (18.61)	7.63 (7.46)	2.58 (2.42)	0.71 (1.15)

APPLICATION TO A FLORIDA LAKE

Lake Eola in downtown Orlando was used to illustrate the application of the model to an actual field situation, a lake with stormwater inputs. The storm drain system collects water from approximately 55 hectares (136 acres) composed of both commercial and residential areas. About 40% of the drainage area is state DOT right of way. The lake is approximately 11 hectares (27 acres). The lake level is generally maintained between 26.5 meters (87.0 feet) and 27 meters (88.5 feet) above sea level. It is a relatively shallow lake with a mean depth of approximately 3 meters. About 73 percent of the lake volume is located with the 3.0 meter fustum layers. Most of the 5000 plus urban lakes in central Florida have similar characteristics. Physical characteristics are shown in Table 9 and a depth map with finite element grid is shown in Figure 26.

TABLE 9

PHYSICAL CHARACTERISTICS FOR LAKE EOLA, FLORIDA

Parameter	Quantity	
Approximate Surface Area	11 hectares	(27 acres)
Approximate Volume	$3.3\ (10^5)m^3$	(8.7×10^6 gallons)
Mean Depth	3.2 meters	(9.92 feet)
Maximum Depth	6.8 meters	(22.3 feet)
Length of Shoreline	1417 meters	(4650 feet)
Number of Stormwater Inputs	6 major, 3 minor	

On Figure 26, superimposed is a finite element grid consisting of 97 triangles. These divisions are more than required to determine profiles of dissolved oxygen deficit; however, for tutorial purposes they were deemed necessary. Each major stormwater input is illustrated by an arrow. Smaller (less volume) inputs exist. Smaller triangles were constructed near each stormwater input. This was done to increase the sensitivity and demonstrate water quality changes, if they exist. The model network is shown in Table 10. To reduce the bandwidth, Robert Smith of the research team wrote a computer program using the ideas of R. J. Collins (1973). The bandwidth was reduced to 25, thus saving computational time. The bandwidth reduction program is shown in Appendix B.

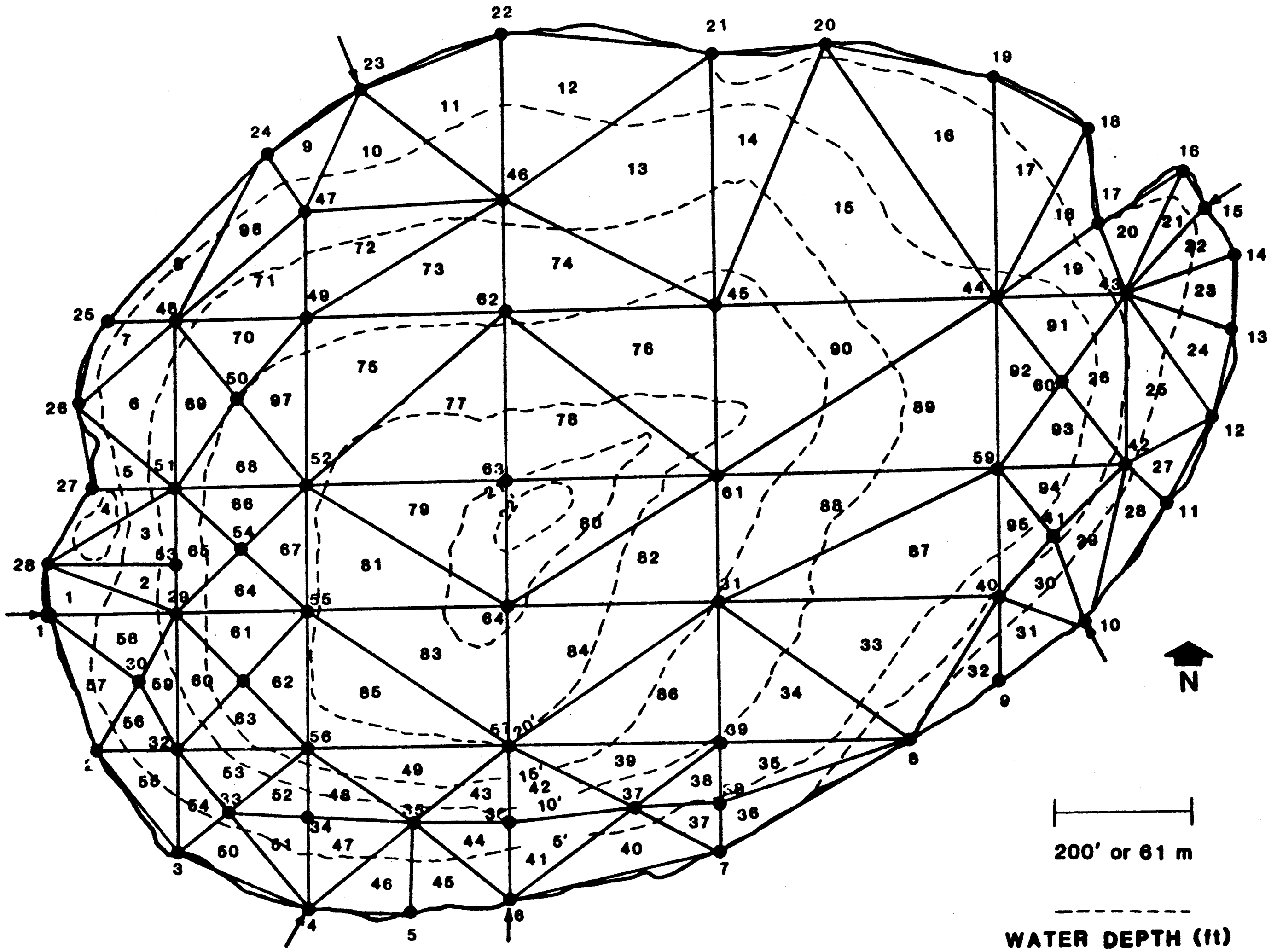

FIGURE 26: DEPTH MAP OF LAKE EOLA.
WITH FINITE ELEMENT GRID

TABLE 10

LAKE EOLA NODAL NETWORK
(MATRIX NOD)

ELEMENT	DESCRIPTION			ELEMENT	DESCRIPTION		
1	1	29	28	51	4	34	33
2	28	29	53	52	33	34	56
3	28	53	51	53	33	56	32
4	28	51	27	54	3	33	32
5	27	51	26	55	3	32	2
6	26	51	48	56	2	32	30
7	48	25	26	57	2	30	1
8	25	48	24	58	30	29	1
9	24	47	23	59	32	29	30
10	47	46	23	60	32	58	29
11	23	46	22	61	58	55	29
12	46	21	22	62	58	56	55
13	46	45	21	63	32	56	58
14	45	20	21	64	29	55	54
15	45	44	20	65	29	54	51
16	44	19	18	66	54	52	51
17	44	18	19	67	54	55	52
18	44	17	18	68	51	52	50
19	44	43	17	69	51	50	48
20	43	16	17	70	50	49	48
21	43	15	16	71	48	49	47
22	43	14	15	72	49	46	47
23	43	13	14	73	49	62	46
24	12	13	43	74	62	45	46
25	42	12	43	75	52	62	49
26	60	42	43	76	62	61	45
27	42	11	12	77	52	63	62
28	10	11	42	78	63	61	62
29	10	42	41	79	52	64	63
30	40	10	41	80	64	61	63
31	9	10	40	81	55	64	52
32	8	9	40	82	64	31	61
33	8	40	31	83	55	57	64
34	39	8	31	84	57	39	64
35	38	8	39	85	56	57	55
36	7	8	38	86	57	39	31
37	7	38	37	87	39	40	59
38	37	38	39	88	39	59	61
39	37	39	47	89	61	59	44
40	6	7	37	90	61	44	45
41	6	37	36	91	60	43	44
42	36	37	57	92	59	60	44
43	35	36	57	93	59	42	60
44	6	36	35	94	41	42	59
45	5	6	35	95	40	41	59
46	4	5	35	96	48	47	24
47	4	34	35	97	50	52	49
48	34	35	56				
49	35	57	56				
50	3	4	33				

Stormwater inputs from a 0.75-hour and a 1.83-hour storm were used to simulate a mixing zone in Lake Eola. These inputs were measured at node 1 shown in Figure 26. These input flow rates, BOD_5, and Kjeldahl nitrogen pollutographs are shown in Figures 27, 28 and 29. The other inputs were scaled based on the watershed area discharging to the lake. The rainfall which produced the hydrographs in Figure 27 was about 2.2 inches. A storm with this volume or greater would occur about 3% of the time.

There are about 120 storms per year in the area of the lake, thus about 3 to 4 storms per year will have a volume equal to or greater than 2.2" and 116 to 117 storms will have a volume equal to or less that 2.2". The 2.2" storm used for simulation is considered to have near maximum potential for dissolved oxygen degradation.

The program LAKE was executed and results indicated a maximum BOD_5 of about 4 mg/l near node 15 which reduced to about 0.4 mg/l after 16 hours from start of storm. Dispersion of BOD_5 was evident with time. The maximum BOD_5 concentration occurred at 2 hours and 20 minutes after the start of the storm. Contours of BOD_5 concentration increases are shown in Figure 30. Background BOD_5 concentration varied between 0.5 and 2.5 mg/l. As expected, higher concentrations were evident around the discharges. The northeast stormwater input (node 15) was about the same magnitude as the eastern stormwater input (node 4); however, the reduced volume for dilution in the area adjacent to node 15 results in a higher concentration near node15.

The BOD_5 values were compared to field collected data and favorable agreement in collected data resulted as shown in Table 11.

TABLE 11

BOD_5 LAKE EOLA CONCENTRATION

(5:00 PM: 3 HOURS AFTER START OF STORM)

BACKGROUND = 2 MG/L (ASSUMED)*

NODE	PREDICTED	MEASURED
46	2.3	2.0
52	2.3	2.0
57	2.3	2.0
60	3.0	3.5
61	2.0	2.0

*Lake measurements 2 weeks before resulted in an average BOD_5 concentration of 2 mg/l.

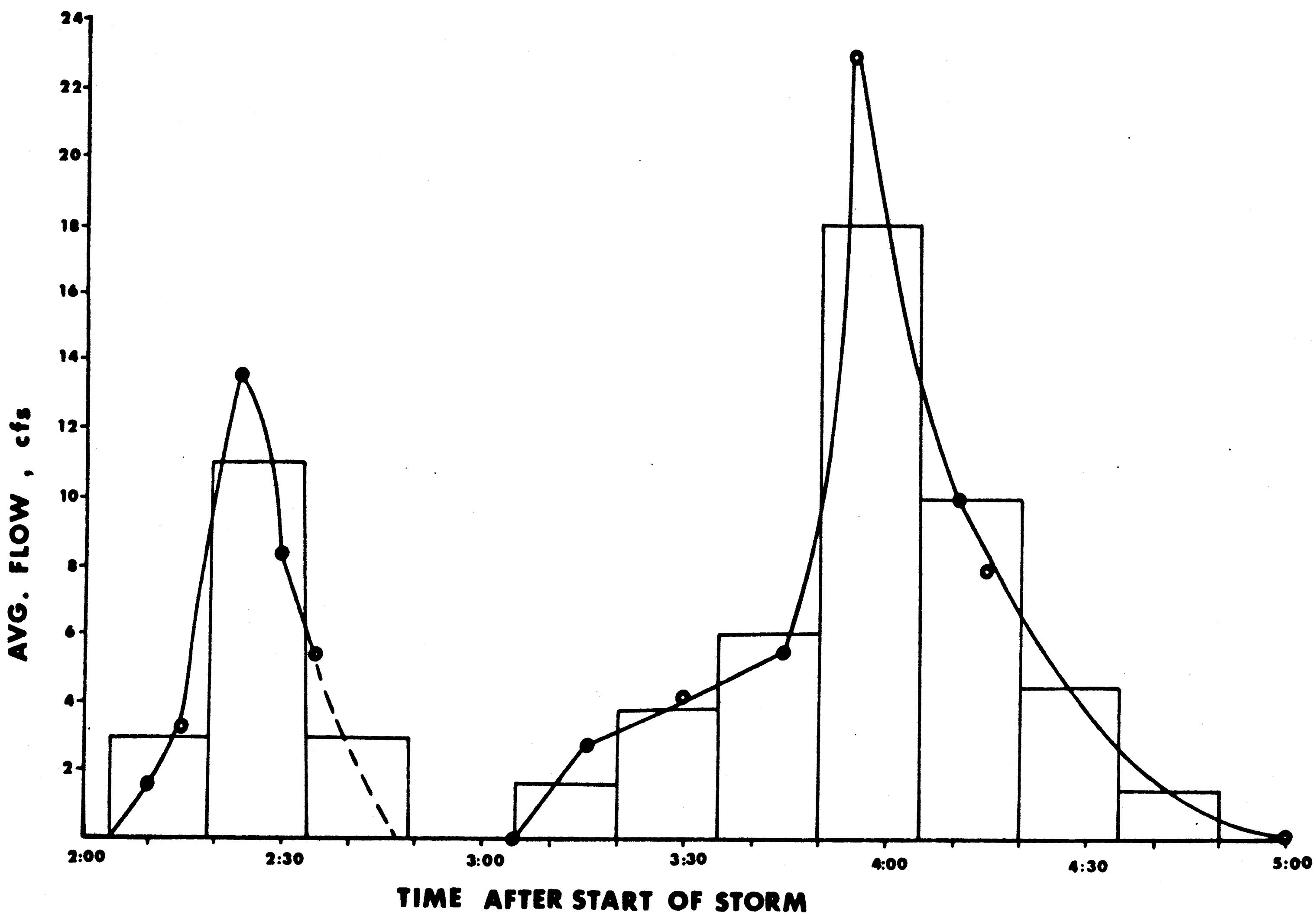

Fig. 27: Hydrographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments.

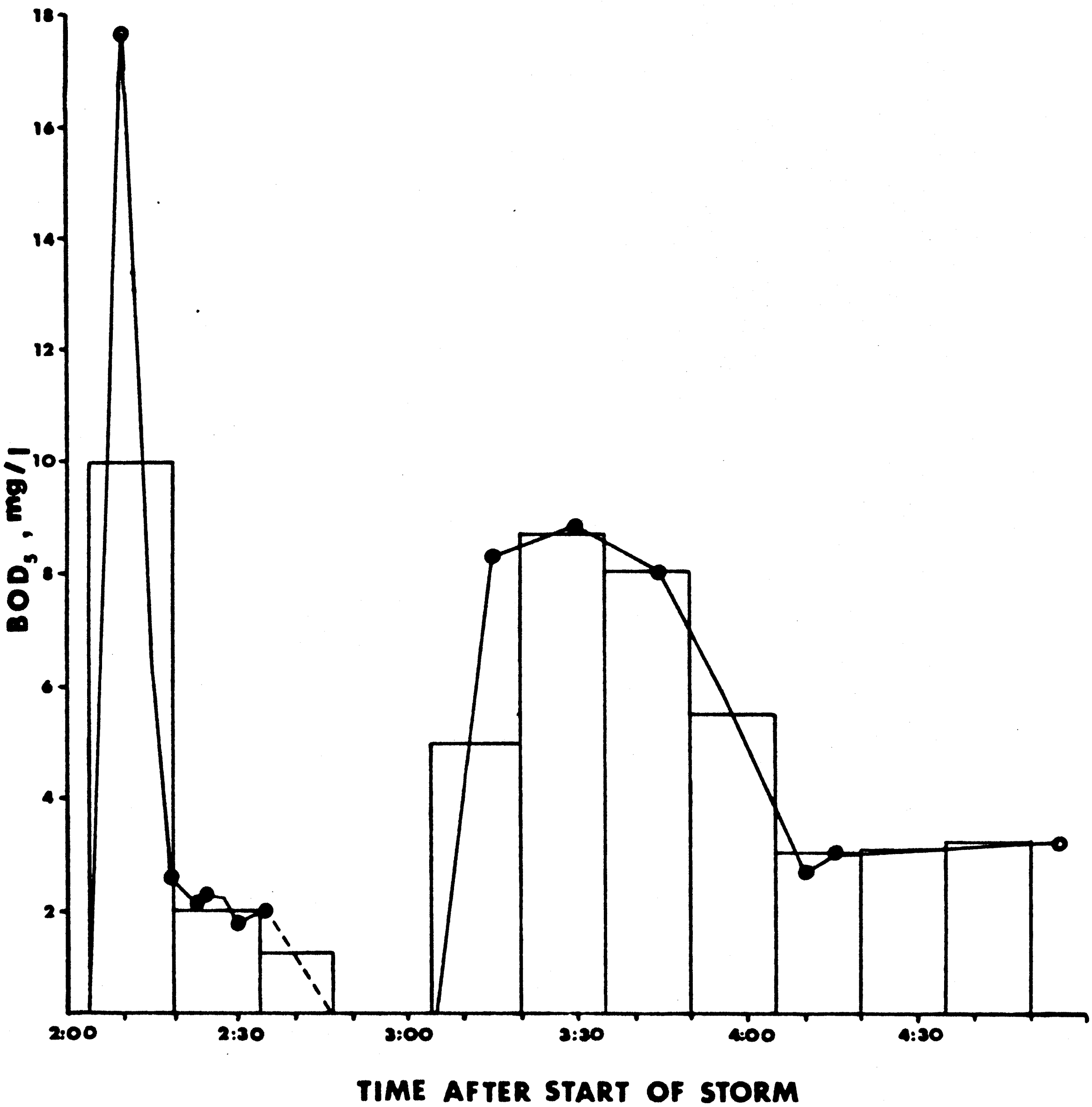

Fig. 28: Carbonaceous BOD_5 pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments.

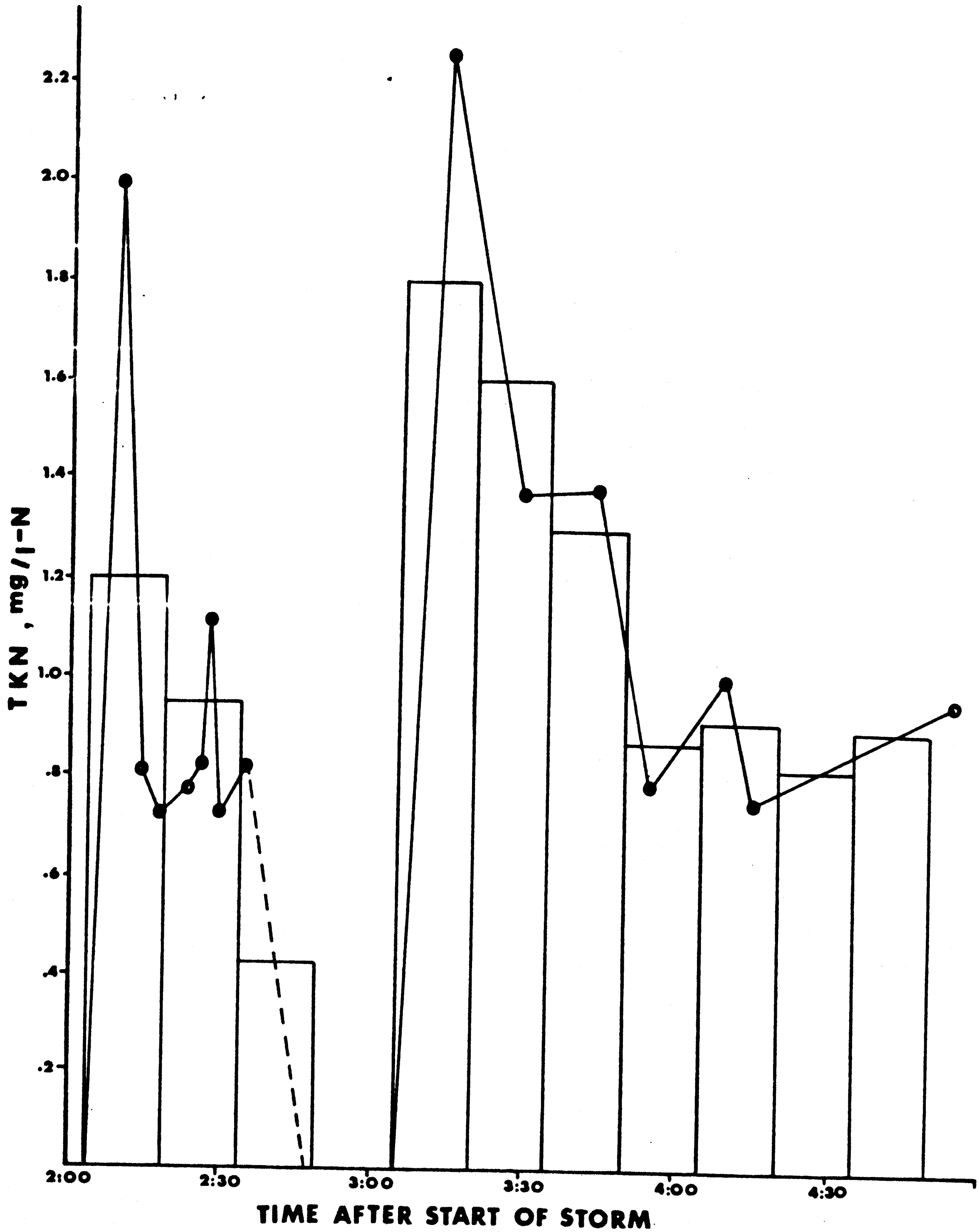

Fig. 29: Total Kjeldahl nitrogen pollutographs associated with the 0.75 hour and 1.83 hour stormwater events divided into 15 minute time increments

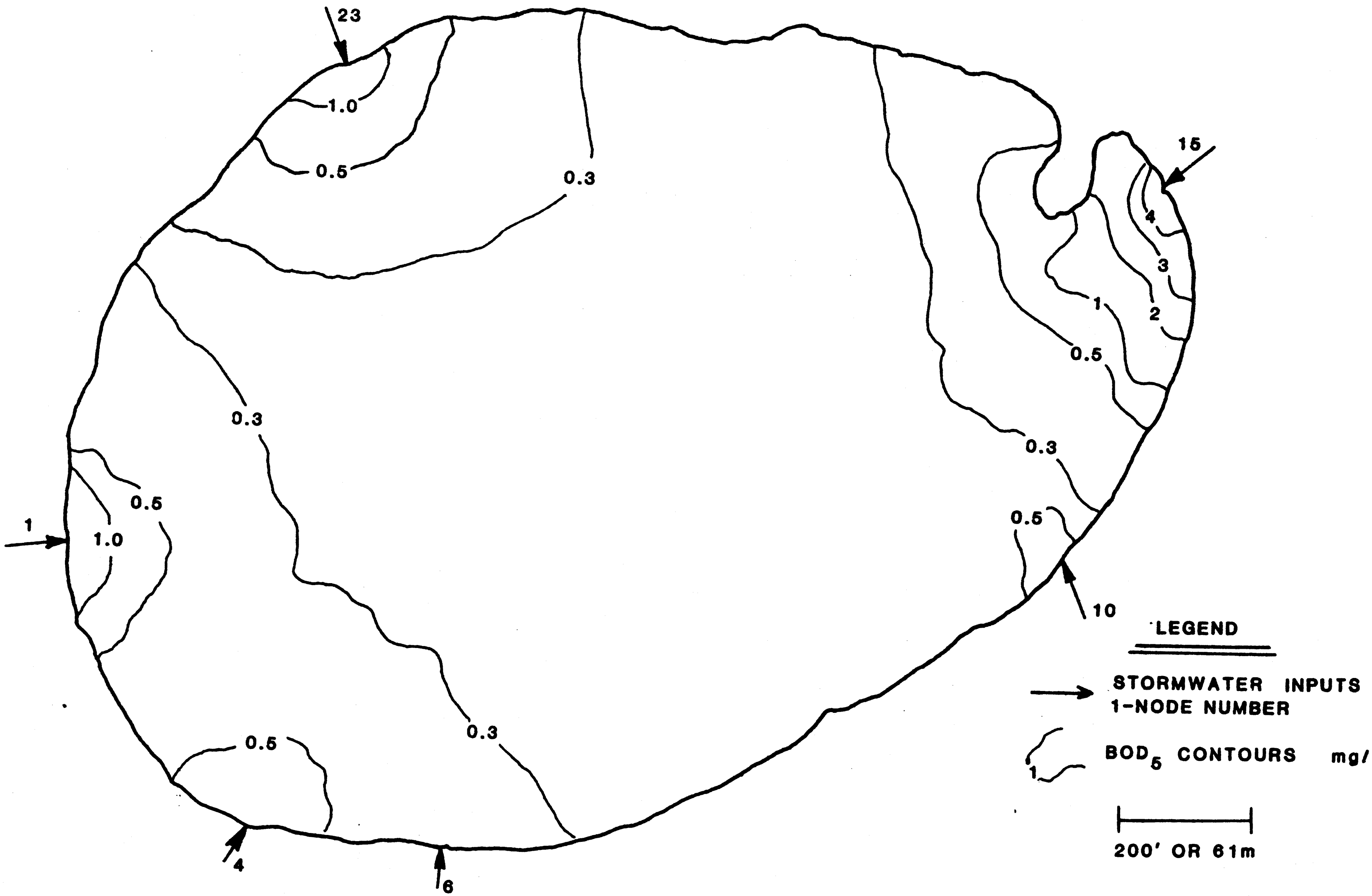

FIGURE 30: MAXIMUM BOD_5 CONCENTRATION CONTOURS RESULTING FROM STORMWATER INPUTS OCCURRING 2 HRS: 20 MIN AFTER START OF STORM

LAKE can be used to conduct a time-dependent simulation of lake concentrations. The accuracy of the simulation depends on the input data estimates. It would be a sizable task to collect field data concurrent with computer program execution. However, this would calibrate and verify the model.

A plot of Dissolved Oxygen Deficit indicates very little (0 to 0.2 mg/l) depletion due to stormwater inputs. As expected, the maximum depletion occurs adjacent to the stormwater discharges. Castro (1982) showed in his work that dissolved oxygen deficit from stormwater was not as significant as that deficit resulting from stormwater overflows. It would appear from this work, that immediate Dissolved Oxygen Depletion (during and a few hours after a storm) is not a major problem in Lake Eola. However, algal activity has been identified as producing a problem with low levels of dissolved oxygen, and algal activity has been related to stormwater discharges. In addition, the metals in stormwater may cause biological decay or death among plant and animal species in a lake. The LAKE model can be modified to incorporate nutrient and metal concentration descriptions. Again, the input data on mass loadings, kinetics and velocity gradients must be defined. At present, the inputs are not defined. Nevertheless, the model is appropriate for lake concentration simulations.

CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

A review of literature on modeling efforts to simulate the assimilative capacities of surface water bodies revealed an extensive deficiency in the assessment of mixing zone requirements for stormwater discharges in aquatic environments.

In general, lower quality characteristics of the wastestream and/or receiving water flow increased the combination of unacceptable operational capacities (volumetric and mass loadings) for the treatment facility in question. Conversely, improved effluent or stream quality increased the combination of acceptable operational capacities, suggesting that the mixing zone length standards could be met at higher effluent discharge capacities and lower stream volumetric flowrates.

For rivers and streams, the surface and cross-sectional area requirements for a mixing zone were plotted as a function of a dilution ratio (effluent flowrate/receiving water flowrate) to investigate the compliance to mixing zone cross-sectional area standards. As expected, the curves developed suggested that both mixing zone cross-sectional and surface area requirements increased as a function of increasing dilution ratios. This is due to the increase in the pollutant spread and depletion of the DO resources in the hypothetical stream stipulated.

It is recommended that:

(1) the effects of these parameters on acceptable and unacceptable regions of operations be analyzed in field applications, and

(2) a mixing zone volume concept be utilized in developing appropriate mixing zone standards.

These recommendations would allow tighter restrictions on stormwater discharges into a stream. Current mixing zone length standards as well as cross-sectional and surface area recommendations can then be combined to assess mixing zone requirements in streams and rivers.

Since SWOPS is a one-dimensional transient model, only the length requirements transient model, only the length requirements for mixing zones were investigated. However, the model was first modified to give physical interpretations of these spatial and temporal distributions for the Lagrange Coordinate System. The modifications for time and distance calculations were not verified, however, subsequent time sensitivity and mixing zone length analysis suggested that the modifications are reasonable.

To investigate the sensitivity of SWOPS to changes in the number of time increments specified for a particular stormwater event, a time sensitivity analysis was conducted on SWOPS. The hydrograph shape characteristics were defined by a dimensionless factor representing the ratio of the second moment to the product of the first moment and the time required to reach the peak discharge. The relationship obtained between the number of time increments required versus the dimensionless ratio was observed to be curvilinear in characteristic.

Since only four points were derived for the curve, it is suggested that more stormwater or combined sewer events be investigated and utilized in a similar analysis. To make the relationship more applicable to field studies, it is recommended that stream characteristics, such as the measured dispersion coefficient, be incorporated in the dimensionless ratio.

The transient nature of stormwater require that mixing zone length requirements for these discharges also be modeled as a function of time. SWOPs was, therefore, utilized to predict the duration and magnitude of violations to the 800 m and 4.0 mg/l length and minimum averaged DO concentration standards, respectively, in mixing zones. The mixing length and DO sag curves developed for two assumed initial DO concentrations in the stream concluded that the magnitude and duration of violations to a mixing zone length, 800 m, is dependent on the initial quality of the stream in question. The higher the quality of the stream, the greater the compliance to the standards. Results suggested stormwater events may have minimal impact on mixing zone requirements in streams or rivers.

It can, therefore, be concluded that TWOD and SWOPS can be incorporated in mixing zone analysis provided that all stream and effluent (point and non-point) qualitative and quantitative parameters effecting stream response can be measured in the field. The limitations associated with each model should be understood and the models should not be applied to existing streams unless the simplifying assumptions utilized in the model development can justifiably be utilized in these streams. The models would have to be calibrated with existing stream data before they can adequately simulate the assimilative capacity of the stream in question.

The time-dependent nature of stormwater also requires that lake mixing zone areas be determined as functions of time. A computer model, called LAKE, was developed to simulate dissolved oxygen and BOD_5 values during and within a few hours to a day after the runoff event. In its present form, LAKE can be used to determine mixing zone requirements based on BOD_5 and dissolved oxygen.

The model results were compared with an analytical solution. The results were similar. Also, the model was used to simulate BOD_5 and Dissolved Oxygen response curves from stormwater inputs into Lake Eola, a 27-acre, urban Florida lake. The BOD_5 results were excellent and compare favorably to field-collected data. Little dissolved oxygen deficit was calculated.

Based on the mixing zone concept as presently enforced, it appears that stormwater has little effect on the immediate dissolved oxygen demand in a lake. Al Castro (1982) came to the same conclusion in his work with mixing zones in rivers and streams. However, it must be emphasized that immediate demands are used. Since other more complex organics exist in stormwater, long-term demands are most likely present but not well documented, and knowledge of long-term dynamics is minimal.

It is recommended that:

1. Field-collected data on the input parameters and output variables of the LAKE model would be beneficial to further document the reliability of the model. For simplicity, this should be done at first for short-term effects assuming minimal internal recycling within the water column.
2. Field investigations should be completed on the kinetics and mass transfer mechanisms of long-term effects of organics, nutrients and metals in stormwaters. The model LAKE was developed to incorporate new field data. Of course, model calibration and verification is a vital part of the field-conducted surveys.
3. Use of the LAKE mixing zone model for BOD_5 and DO should be encouraged for short-term (hours to a few days) effects. Results can be used for environmental impact and planning purposes. Results could be used to specify the quantity and quality of stormwater discharges into a lake.

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APPENDIX A

LAKE PROGRAM LISTING

INPUT VARIABLES
FOR
FINITE ELEMENT LAKE MODEL

NNOD	number of nodal points
NELM	number of elements
C(I)	concentration of pollutant at I'th node, gm/m^3
D(I)	concentration of dissolved oxygen deficit at I'th node, gm/m^3
P(I)	active node indicator; 1 for active node 0 for inactive at I'th node
X(I)	X coordinate of I'th node, meters
Y(I)	Y coordinate of I'th node, meters
H(I)	water depth at I'th node, meters
UX(I)	X component of water velocity at I'th node, m/sec
UY(I)	Y component of water velocity at I'th node, m/sec
ANGLE	angle between velocity vector and positive X axis, degrees
DT	time increment, seconds
MT	number of time increments in integration, number
KR	biodegradation constant, sec^{-1}
KD	deoxygenation constant, sec^{-1}
KL	reaeration constant, sec^{-1}
ELONG	longitudinal dispersion coefficient, m^2/sec
ELAT	lateral dispersion coefficient, m^2/sec
NOPS	number of active point sources
NOD(I,J)	a matrix showing the relationship between node and element numbers
PDATA (I,13)	a vector specifying point sources: first element is node number, second element is start time (sec), third element is stop time (sec), the remaining 10 elements are rates of discharge of pollutants in grams/sec over each of 10 time increments.

```
C     DISPERSION MODEL
C     WRITTEN BY MR. BOB SMITH     CEES DEPARTMENT                PAGE 0001
C     FORMAT CHANGES WERE MADE
C     NNOD=NUMBER OF NODES
C     NELM=NUMBER OF ELEMENTS
C     ELONG=LONGITUDINAL DISPERSION IN SQM/HR
C     ELAT=LATITUDINAL DISPERSION IN SQM/HR
C     DT=POLLUTANT INPUT INTERVAL(SECONDS)
C     MT=NUMBER OF TIME INTEGRATIONS
C     NOPS=NUMBER OF POINT SOURCES
C     KR=BIODEGRADATION RATE(BOD BOTTLE/HR)
C     KD=DE-OXYGENATION RATE( /HR)
C     KL=RE-AERATION RATE( /HR)
C     ANGLE=VELOCITY ANGLE
C     C(I)=INITIAL POLLUTANT CONCENTRATION AT I'TH NODE(MG/L)
C     D(I)=INITIAL DISSOLVED OXYGEN DEFICIT AT I'TH NODE(MG/L)
C     P(I)=ACTIVE SOURCE INDICATOR ;USE 1. FOR ACTIVE NODE,0 OTHERWISE
C     X(I)=X COORDINATE OF THE I'TH NODE IN METERS(M)
C     Y(I)=Y COORDINATE OF THE I'TH NODE(M)
C     H(I)=WATER DEPTH AT THE I'TH NODE(M)
C     UX(I)=COMPONENT OF WATER VELOCITY IN THE X DIRECTION AT I'TH NODE(M/HR)
C     UY(I)=COMPONENT OF WATER VELOCITY IN THE Y DIRECTION AT I'TH NODE(M/HR)
C     SET UP DIMENSIONS
      DIMENSION C(100),X(100),Y(100),NOD(100,3),
     1P(100),AREA(100),H(100),PDATA(5,13),BD(100),ARRAY(100,100),D(100),
     2FMATX(100,100),DX(3),DY(3),FMATX(100,100),UX(100),
     3UY(100),HUX(3),HUY(3),HBAR(100)
      COMMON AMATX(100,100),B(100)
      REAL KR,KD,KL
C     READ NUMBER OF NODES AND NUMBER OF ELEMENTS
C     READ INPUT VARIABLES
      READ(5,1)NNOD,NELM
    1 FORMAT(I3,1X,I3)
      READ(5,12)ELONG,ELAT,DT,MT,NOPS,KR,KD,KL,ANGLE
   12 FORMAT(3F10.3,1X,I2,1X,I2,1X,2F5.3,2F5.2)
      READ(5,2)(C(I),D(I),P(I),X(I),Y(I),H(I),UX(I),UY(I),I=1,NNOD)
    2 FORMAT(8F10.2)
      READ(5,3)((NOD(I,J),J=1,3),I=1,NELM)
    3 FORMAT(I2,1X,I2,1X,I2)
      READ(5,23)((PDATA(I,J),J=1,13),I=1,NOPS)
   23 FORMAT(3F5.0,10F6.0)
C     CHECK INPUT
      WRITE(6,50)
   50 FORMAT(1X,'*********************'/1X,
     1'**** INPUT DATA ****'/1X,
     2'*********************'/)
      WRITE(6,29)ELONG,ELAT,DT,MT,NOPS,KL,KD,KR,ANGLE
   29 FORMAT(1X,'LONGITUDINAL DISPERSION = ',F10.3,' SQ METERS/SEC'/
     11X,'LATITUDINAL DISPERSION = ',F10.3,' SQ METERS/SEC'/
     31X,'TIME INTERVAL BETWEEN ITERATIONS = ',F10.3/
     21X,'NUMBER OF ITERATIONS = ',I3,/
     41X,'NUMBER OF POINT SOURCES = ',I2/
     51X,'KL = ',F8.4/1X,'KD = ',F8.4/1X,'KR = ',F8.4/
     61X,'ANGLE = ',F8.4,' DEGREES'////)
      WRITE(6,53)
   53 FORMAT(/1X,'**********************************'/1X,
     1'**** PHYSICAL SYSTEM SET-UP ****'/1X,
     2'**********************************'/)
      WRITE(6,6)
    6 FORMAT(////1X,'STORMWATER',T15,'POLLUTANT',T31,'CONCENTRATION',T47
     1,'DISSOLVED',T63,'X-COORDINATE',T79,'Y-CORRDINATE',T94,'DEPTH',
     2T104,'X-VELOCITY',T117,'Y-VELOCITY',/,
     33X,'NODE',T15,'INPUT',T31,'(GRAMS/CUBIC',T47,'OXYGEN',T63,
     4'(METERS)',T79,'(METERS)',T94,'(METERS)',T104,'(METERS/',T117,
     5'(METERS/',/,
     6T15,'(GRAMS/SEC)',T38,'METER)',T47,'GRAMS/CUBIC',T109,'SEC)',
     7T124,'SEC)',/,T53,'METER)',//)
      WRITE(6,5)(I,P(I),C(I),D(I),X(I),Y(I),H(I),UX(I),UY(I),I=1,NNOD)
    5 FORMAT(3X,I3,T15,F10.4,T31,F10.4,T47,F10.4,T63,F10.4,T79,F10.4,
     1T94,F8.4,T104,F10.4,T117,F10.4)
      DO 31 I=1,NOPS
      WRITE(6,25)
   25 FORMAT(///1X,'STORMWATER',T15,'STARTING TIME',T36,'ENDING TIME'/
     13X,'NODE',T18,'(SEC)',T36,'(SEC)')
      WRITE(6,24)(PDATA(I,J),J=1,3)
   24 FORMAT(1X,F6.3,T18,F6.3,T36,F6.3//)
      WRITE(6,51)
   51 FORMAT(/1X,'POLLUTANT DATA (GRAMS/SEC FOR 10 INPUT TIME INCREMENTS
```

```
      1)'/)
      WRITE(6,26)(PDATA(I,J),J=4,13)
   26 FORMAT(10(4X,F6.3)//)
   31 CONTINUE
      B2=0.0
      DO 19 I=1,NELM
      K1=NOD(I,1)
      K2=NOD(I,2)
      K3=NOD(I,3)
      K12=(K1-K2)**2
      K13=(K1-K3)**2
      K23=(K2-K3)**2
      IF(K12-B2)14,14,13
   13 B2=K12
   14 IF(K13-B2) 17,17,16
   16 B2=K13
   17 IF(K23-B2)19,19,18
   18 B2=K23
   19 CONTINUE
      MS=B2**.5
      NBAND=2*MS+1
C     GENERATE INTIAL CONDITIONS
   11 EXX=ELONG*(COS(ANGLE))**2+ELAT*(SIN(ANGLE))**2
      EYY=ELONG*(SIN(ANGLE))**2+ELAT*(COS(ANGLE))**2
      EXY=(ELONG-ELAT)*SIN(ANGLE)*COS(ANGLE)
      WRITE(6,4) NBAND
    4 FORMAT(////1X,'BANDWIDTH =',I3///)
      TIME=0.
      MS=(NBAND+1)/2
      DO 30 I=1,NNOD
      B(I)=0.
      BD(I)=0.
      DO 30 J=1,NNOD
      ARRAY(I,J)=0.
      EMATX(I,J)=0.
      FMATX(I,J)=0.
   30 CONTINUE
      DO 60 I=1,NELM
      K1=NOD(I,1)
      K2=NOD(I,2)
      K3=NOD(I,3)
      HBAR(I)=(H(K1)+H(K2)+H(K3))/3.
      AREA(I)=(X(K1)*Y(K2)-X(K2)*Y(K1))/2.+(X(K2)*Y(K3)-X(K3)*Y(K2))/2.+
     1(X(K3)*Y(K1)-X(K1)*Y(K3))/2.
      DX(1)=(Y(K2)-Y(K3))/2./AREA(I)
      DX(2)=(Y(K3)-Y(K1))/2./AREA(I)
      DX(3)=(Y(K1)-Y(K2))/2./AREA(I)
      DY(1)=(X(K3)-X(K2))/2./AREA(I)
      DY(2)=(X(K1)-X(K3))/2./AREA(I)
      DY(3)=(X(K2)-X(K1))/2./AREA(I)
      HF1=H(K1)/10.+H(K2)/30.+H(K3)/30.
      HF2=H(K1)/30.+H(K2)/30.+H(K3)/60.
      HF3=H(K1)/30.+H(K2)/60.+H(K3)/30.
      HUX(1)=UX(K1)*HF1+UX(K2)*HF2+UX(K3)*HF3
      HUY(1)=UY(K1)*HF1+UY(K2)*HF2+UY(K3)*HF3
      HF1=H(K1)/30.+H(K2)/30.+H(K3)/60.
      HF2=H(K1)/30.+H(K2)/10.+H(K3)/30.
      HF3=H(K1)/60.+H(K2)/30.+H(K3)/30.
      HUX(2)=UX(K1)*HF1+UX(K2)*HF2+UX(K3)*HF3
      HUY(2)=UY(K1)*HF1+UY(K2)*HF2+UY(K3)*HF3
      HF1=H(K1)/30.+H(K2)/60.+H(K3)/30.
      HF2=H(K1)/60.+H(K2)/30.+H(K3)/30.
      HF3=H(K1)/30.+H(K2)/30.+H(K3)/10.
      HUX(3)=UX(K1)*HF1+UX(K2)*HF2+UX(K3)*HF3
      HUY(3)=UY(K1)*HF1+UY(K2)*HF2+UY(K3)*HF3
      DO 60 J=1,3
      LR=NOD(I,J)
      DO 60 K=1,3
      LC=NOD(I,K)+MS-LR
      IF(LC-MS)66,67,66
   66 ARRAY(LR,LC)=ARRAY(LR,LC)+AREA(I)/12.*HBAR(I)
      GO TO 68
   67 ARRAY(LR,LC)=ARRAY(LR,LC)+AREA(I)/6.*HBAR(I)
   68 EMATX(LR,LC)=EMATX(LR,LC)+(DX(K)*HUX(J)+DY(K)*HUY(J))*AREA(I)
      FMATX(LR,LC)=FMATX(LR,LC)+HBAR(I)*AREA(I)*(EXX*DX(K)*DX(J)+
     1EYY*DY(K)*DY(J)+EXY*(DX(K)*DY(J)+DX(J)*DY(K)))
   60 CONTINUE
      WRITE(6,7)
```

```
 7 FORMAT(/1X,'ELEMENT',T19,'NODAL',T42,'AREA',T62,'HBAR'/
  12X'NUMBER',T19,'DESCRIPTION',T42,'(SQ METERS)',T62,'(METERS)'/)
    DO 70 I=1,NELM
    WRITE(6,15)I,(NOD(I,J),J=1,3),AREA(I),HBAR(I)
15  FORMAT(3X,I2,T19,I2,1X,I2,1X,I2,T42,F10.4,T59,F10.4)             PAGE 0003
70  CONTINUE
    CALL PRNT(NNOD,ARRAY)
    CALL PRNT(NNOD,EMATX)
    CALL PRNT(NNOD,FMATX)
    WRITE(6,52)
52  FORMAT(/1X,'****************'/1X,
   1'**** OUTPUT ****'/1X,
   2'****************'////)
    DO 500 NT=1,MT
    TIME=TIME+DT
    WRITE(6,27)TIME
27  FORMAT(///1X,'TIME = ',F10.2,' SEC'///)
    DO 20 I=1,NNOD
    B(I)=0.
    BD(I)=0.
    DO 20 J=1,NNOD
    AMATX(I,J)=0.0
20  CONTINUE
    DO 40 I=1,NOPS
    J=PDATA(I,1)
    NSTRT=PDATA(I,2)/DT
    NEND=PDATA(I,3)/DT
    IF(MT-NEND)39,39,32
32  IF(NT-NSTRT)40,34,34
34  IF(NT-NEND)35,35,40
35  K=NT-NSTRT+3
    B(J)=PDATA(I,K)
    P(J)=PDATA(I,K)
    GO TO 40
39  B(J)=PDATA(I,4)
    P(J)=PDATA(I,4)
40  CONTINUE
    GAUSS ELIMINATION
    DO 200 I=1,NNOD
    DO 200 J=1,NBAND
    K=J+I-MS
    IF(K) 230,230,220
220 IF(K-NNOD) 225,225,230
225 B(I)=B(I)+(ARRAY(I,J)*(1./DT-KR/2)-FMATX(I,J)/2.-EMATX(I,J)/2.)*
   1C(K)
230 AMATX(I,J)=ARRAY(I,J)*(1./DT+KR/2.)+FMATX(I,J)/2.+EMATX(I,J)/2.
200 CONTINUE
    CALL SLBNS(NNOD,NBAND,NNOD,NBAND)
    DO 330 I=1,NNOD
    C(I)=0.0
330 CONTINUE
    DO 340 I=1,NNOD
    C(I)=B(I)
340 CONTINUE
    DO 210 I=1,NNOD
    DO 210 J=1,NBAND
    K=J+I-MS
    IF(K) 209,209,207
207 IF(K-NNOD) 208,208,209
208 BD(I)=BD(I)+KD*ARRAY(I,J)*C(K)+(ARRAY(I,J)*(1./DT-KL/2.)-
   1FMATX(I,J)/2.-EMATX(I,J)/2.)*D(K)
    B(I)=BD(I)
209 AMATX(I,J)=(1./DT+KL/2)*ARRAY(I,J)+FMATX(I,J)/2.+EMATX(I,J)/2.
210 CONTINUE
    CALL SLBNS(NNOD,NBAND,NNOD,NBAND)
    DO 350 I=1,NNOD
    D(I)=0.0
350 CONTINUE
    DO 360 I=1,NNOD
    D(I)=B(I)
360 CONTINUE
    PRINT A AND B MATRIX SOLUTION
    WRITE(6,303)
303 FORMAT(1X,'STORMWATER',T20,'POLLUTANT',T40,'CONCENTRATION',T61,
   1'DISSOLVED'/
   23X,'NODE',T20,'INPUT',T40,'(GRAMS/CUBIC',T61,'OXYGEN'/
   3T20,'(GRAMS/SEC)',T40,'            METER)',T61,'(GRAMS/CUBIC METER)'/)
    DO 310 I=1,NNOD
```

```
310 WRITE(6,22)I,P(I),C(I),D(I)
 22 FORMAT(3X,I3,T18,F10.3,T40,F20.3,T62,F20.3)
500 CONTINUE
    DEBUG SUBCHK
    STOP
    END
```

```
      SUBROUTINE SLBNS(N,M,NX,MX)

      SOLUTION OF LINEAR SYSTEMS OF EQUATIONS
      BY THE GAUSS ELIMINATION METHOD, FOR
      NON SYMETRIC BANDED SYSTEMS

      A- CONTAINS THE SYSTEM MATRIX, STORED
      ACCORDING TO THE NON SYMETRIC
      BANDED SCHEME
      ARRAY B CONTAINS THE INDEPENDENT COEFFICIENTS.
      AFTER SOLUTION IT CONTAINS THE UNKNOWN VALUES.

      N IS THE NUMBER OF UNKNOWNS
      M IS THE BANDWITH
      NX IS THE ROW DIMENSIONS OF ARRAYS A AND B
      MX IS THE COLUMN DINENSIONS OF ARRAY A

      COMMON AMATX(100,100),B(100)
      N1=N-1
      MS=(M+1)/2
      DO 100 K=1,N1
      C=AMATX(K,MS)
      K1=K+1
      IF(ABS(C)-0.000001)1,1,3
    1 WRITE(6,2) K
    2 FORMAT(' **** SINGULARITY IN ROW',I5)
      GO TO 300

      DIVIDE ROW BY DIAGONAL COEFFICIENT

    3 NI=K1+MS-2
      IF(NI-N)6,6,7
    6 L=NI
      GO TO 11
    7 L=N
   11 DO 4 J=K1,L
      K2=MS+J-K
    4 AMATX(K,K2)=AMATX(K,K2)/C
      B(K)=B(K)/C

      ELIMINATE UNKNOWN X(K) FROM ROW I

      DO 10 I=K1,L
      K2=MS+K-I
      C=AMATX(I,K2)
      DO 5 J=K1,L
      K2=MS+J-I
      K3=MS+J-K
    5 AMATX(I,K2)=AMATX(I,K2)-C*AMATX(K,K3)
   10 B(I)=B(I)-C*B(K)
  100 CONTINUE

      COMPUTE LAST UNKNOWN

      IF(ABS(AMATX(N,MS))-0.000001)1,1,101
  101 B(N)=B(N)/AMATX(N,MS)

      APPLY BACKSUBSTITUTION PROCESS TO COMPUTE REMAINING UNKNOWNS

      DO 200 I=1,N1
      K=N-I
      K1=K+1
      NI=K1+MS-2
      IF(NI-N)8,8,9
    8 L=NI
      GO TO 12
    9 L=N
   12 DO 200 J=K1,L
      K2=MS+J-K
  200 B(K)=B(K)-AMATX(K,K2)*B(J)
  300 RETURN
      END
```

```
      SUBROUTINE PRNT(NNOD,AMATX)
      DIMENSION AMATX(100,100)
      N=NNOD
      L=1
      M=9
   45 IF(N-9)55,55,40
   40 WRITE(6,14)(J,J=L,M)
   14 FORMAT(1H1,9(11X,I2)/)
      DO 50 I=1,NNOD
   50 WRITE(6,6)(AMATX(I,J),J=L,M)
      N=N-9
      L=M+1
      M=M+9
      GO TO 45
   55 WRITE(6,14)(J,J=L,NNOD)
      DO 60 I=1,NNOD
   60 WRITE(6,6)(AMATX(I,J),J=L,NNOD)
    6 FORMAT(9(2X,E11.4))
      RETURN
      END
```

APPENDIX B

NODE RENUMBERING PROGRAM

```
10 REM PROGRAM (RELABEL) FOR RENUMBERING NODES FOR MIN BANDWIDTH
20 READ NNOD,NELM
30 DATA 64,97
40 DIM NOD(100,3),NSUR(70,10),NNEW(70),NOLD(70),RELAB(70)
50 FOR I=1 TO NELM : FOR J=1 TO 3
60 READ NOD(I,J) : NEXT J : NEXT I
70 DATA 1,29,28,28,29,53,28,53,51,28,51,27,27,51,26,26,51,48,48,25,26
80 DATA 25,48,24,24,47,23,47,46,23,23,46,22,46,21,22,46,45,21,45,20,21
90 DATA 45,44,20,44,19,20,44,18,19,44,17,18,44,43,17,43,16,17,43,15,16
100 DATA 43,14,15,43,13,14,12,13,43,42,12,43,60,42,43,42,11,12,10,11,42
110 DATA 10,42,41,40,10,41,9,10,40,8,9,40,8,40,31,39,8,31,38,8,39,7,8,38
120 DATA 7,38,37,37,38,39,37,39,57,6,7,37,6,37,36,36,37,57,35,36,57,6,36
130 DATA 35,5,6,35,4,5,35,4,34,34,34,35,56,35,57,56,3,4,33,4,34,33,33,34
140 DATA 56,33,56,32,3,33,32,3,32,2,2,32,30,2,30,1,30,29,1,32,29,30,32,58
150 DATA 29,58,55,29,58,56,55,32,56,58,29,55,54,29,54,51,54,52,51,54,55,52
160 DATA 51,52,50,51,50,48,50,49,48,48,49,47,49,46,47,49,62,46,62,45,46,52
170 PRINT"ELEMENT NO.   ";
180 DATA 62,49,62,61,45,52,63,62,63,61,62,52,64,63,64,61,63,55,64,52,64,31
190 DATA 61,55,57,64,57,31,64,56,57,55,57,39,31,31,40,59,31,59,61,61,59,44
200 DATA 61,44,45,60,43,44,59,60,44,59,42,60,41,42,59,40,41,59,48,47,24,50
210 DATA 52,49
220 PRINT"  NODE 1  ";
230 PRINT"  NODE 2  ";
240 PRINT"  NODE 3  "
250 FOR I=1 TO NELM
260 PRINT I,NOD(I,1),NOD(I,2),NOD(I,3)
270 NEXT I
280 STOP
290 FOR I=1 TO NNOD : FOR J=1 TO 10
300 NSUR(I,J)=0 : NEXT J : NEXT I
310 FOR I=1 TO NELM : FOR J=1 TO 3
320 LR=NOD(I,J)
330 FOR JJ=1 TO 3 : IF JJ=J THEN 400 ELSE 340
340 NMEN=NOD(I,JJ)
350 FOR K=2 TO 10 :IF NSUR(LR,K)=0 THEN LC=K : GOTO 390
360 IF ABS(NMEN-NSUR(LR,K))=0 THEN 400
370 PRINT LR,LC,NMEN
380 NEXT K
390 NSUR(LR,LC)=NMEN
400 NEXT JJ
410 NEXT J
420 NEXT I
430 STOP
440 IDIFF=0 : FOR I=1 TO NNOD : NMAX=0 : NMIN=10
450 FOR J=2 TO 10 : IF NSUR(I,J)>0 THEN 460 ELSE 490
460 NSUR(I,1)=NSUR(I,1)+1
470 IF NSUR(I,J)>NMAX THEN NMAX=NSUR(I,J)
480 IF NSUR(I,J)<NMIN THEN NMIN=NSUR(I,J)
490 NEXT J
```

```
500 DIFF=NMAX-NMIN
510 IF DIFF>IDIFF THEN IDIFF=DIFF
520 NEXT I
530 FOR I=1 TO NNOD : LPRINT USING "###.    ";I,
540 LPRINT USING "###.    ";NSUR(I,1),NSUR(I,2),NSUR(I,3),NSUR(I,4),NSUR(I,5)
550 LPRINT USING "###.    ";NSUR(I,6),NSUR(I,7),NSUR(I,8),NSUR(I,9),NSUR(I,10
560 NEXT I
570 LPRINT"    "
580 NBAND= IDIFF+1
590 LPRINT"BANDWIDTH EQUALS ";NBAND
600 MAXDIFF=0 : MINDIFF=IDIFF : MINMAX=IDIFF
610 FOR IK=1 TO NNOD
620 IF IK=1 THEN 650
630 IF MAXDIFF<MINMAX THEN 640 ELSE 650
640 MINMAX=MAXDIFF : FOR M=1 TO NNOD : RELAB(M)=NNEW(M) : NEXT M
650 FOR J=1 TO NNOD : NNEW(J)=0 : NOLD(J)=0 : NEXT J
660 I=1 : NOLD(1)=IK : NNEW(IK)=1 : K=1
670 LTEMP=NOLD(I) : K4=NSUR(LTEMP,1)+1 : MAXDIFF=0
680 MAX=NNEW(LTEMP) : MIN=NNEW(LTEMP)
690 IF K4=0 THEN 910
700 FOR JJ=2 TO K4 : K5=NSUR(LTEMP,JJ)
710 IF NNEW(K5)>0 THEN 760
720 K=K+1
730 NNEW(K5)=K : NOLD(K)=K5
740 IF NNEW(K5)>MAX THEN MAX=NNEW(K5)
750 IF NNEW(K5)<MIN THEN MIN=NNEW(K5)
760 NEXT JJ
770 DIFF=MAX-MIN
780 IF DIFF>MAXDIFF THEN MAXDIFF=DIFF
790 IF DIFF<MINDIFF THEN MINDIFF=DIFF
800 PRINT"MAX = ";MAX
810 PRINT"MIN = ";MIN
820 PRINT"DIFF= ";DIFF
830 PRINT"MAXDIFF= ";MAXDIFF
840 PRINT"MINDIFF= ";MINDIFF
850 PRINT"MINMAX= ";MINMAX
860 PRINT"I= ";I
870 PRINT"IK= ";IK
880 FOR M=1 TO NNOD
890 PRINT NNEW(M),NOLD(M) : NEXT M
910 IF K=NNOD THEN 950
920 IF MAXDIFF=>MINMAX THEN 950
930 I=I+1
940 GOTO 670
950 NEXT IK
960 NBAND=MINMAX+1
970 LPRINT"BANDWIDTH EQUALS ";NBAND
980 FOR M=1 TO NNOD
990 LPRINT M,RELAB(M) : NEXT M
1000 END
```

1	61	50	58
2	58	50	59
3	58	59	48
4	58	48	60
5	60	48	53
6	53	48	43
7	43	52	53
8	52	43	42
9	42	32	34
10	32	23	34
11	34	23	35
12	23	24	35
13	23	15	24
14	15	16	24
15	15	7	16
16	7	17	18
17	7	18	17
18	7	8	18
19	7	3	8
20	3	9	8
21	3	10	9
22	3	6	10
23	3	2	6
24	1	2	3
25	4	1	3
26	11	4	3
27	4	5	1
28	12	5	4
29	12	4	13
30	20	12	13
31	21	12	20
32	29	21	20
33	29	20	28
34	22	29	28
35	30	29	22
36	39	29	30
37	39	30	31
38	31	30	22
39	31	22	32
40	40	39	31
41	40	31	41
42	41	31	33
43	44	41	33
44	40	41	44
45	51	40	44
46	54	51	44
47	54	44	55
48	55	44	45
49	44	33	45
50	63	54	56
51	54	55	56
52	56	55	45
53	56	45	57
54	63	56	57
55	63	57	64
56	64	57	62
57	64	62	61
58	62	50	61
59	57	50	62
60	57	49	50
61	49	38	50
62	49	45	38
63	57	45	49
64	50	38	47
65	50	47	48
66	47	37	48
67	47	38	37
68	48	37	46
69	48	46	43
70	46	36	43
71	43	36	32
72	36	23	32
73	36	25	23
74	25	15	23
75	37	25	36
76	25	19	15
77	37	26	25
78	26	19	25
79	37	27	26
80	27	19	26
81	38	27	37
82	27	28	19
83	38	33	27
84	33	22	27
85	45	33	38
86	33	22	28
87	22	20	14
88	22	14	19
89	19	14	7
90	19	7	15
91	11	3	7
92	14	11	7
93	14	4	11
94	13	4	14
95	20	13	14
96	43	32	42
97	46	37	36

BANDWIDTH EQUALS 25

1	61
2	64
3	63
4	54
5	51
6	40
7	39
8	29
9	21
10	12
11	5
12	1
13	2
14	6
15	10
16	9
17	8
18	18
19	17
20	16
21	24
22	35
23	34
24	42
25	52
26	53
27	60
28	58
29	50
30	62
31	28
32	57
33	56
34	55
35	44
36	41
37	31
38	30
39	22
40	20
41	13
42	4
43	3
44	7
45	15
46	23
47	32
48	43
49	36
50	46
51	48
52	37
53	59
54	47
55	38
56	45
57	33
58	49
59	14
60	11
61	19
62	25
63	26
64	27

APPENDIX C

INTEGRATION OF THE GOVERNING EQUATIONS

INTEGRATION OF THE GOVERNING EQUATIONS

The Galerkin finite element method is used to solve the governing equations for C and D in the spatial dimensions, and an implicit method is used for integration in the time domain. The central idea behind the finite element method of numerical integration is to allow the dependent variables to be represented by approximating functions containing unknown parameters, to substitute the approximating functions into the governing partial differential equations, and the to integrate the governing equations over the domain of the independent variables to solve for the values of the parameters which allow the approximating functions to best satisfy the governing equations. Since a single approximating function can usually not be expected to adequately represent the dependent variable over the whole domain of independent variables, the domain of independent variables is divided into discrete segments called elements. For the lake model, this amounts to dividing the lake or river surface into a finite number of triangular elements. Extreme points of the elements are called nodes and both the elements and nodes are numbered for identification within the computational process.

If the triangular elements are made sufficiently small, a planar approximating function can be visualized as being the sum of three simpler planar functions, sometimes called basis functions. One basis function is associated with each of the three nodes at the extreme points of each element. The planar basis function has a value of one at the node with which it is associated and values of zero at the other two nodes. The symbol u_i is used to designate a basis function where i is the number of the node with which it is associated. Thus, the pollutant concentration can be represented over a single element by the equation: $C = C_i u_i + C_j u_j + C_k u_k$ where C_i is the pollutant concentration at the i'th node. Notice that the value of the sum of the three basis functions for any element is one at all points within the element.

In the Galerkin version of the finite element method each term of the governing equation is weighted with (multiplied by) the basis function, $U = u_i + u_j + u_k$, before the integrations are performed. More specifically, the governing equation is first multipled by u_i and integrated, then by u_j and integrated, and finally by u_k and integrated. Thus, integration of the governing equation over a single element, after approximating functions have been substituted for the dependent variables, yields three independent relationships; one for each of the three nodes. Integrations are performed on an element-by-element basis and the resulting linear relationships are later assembled into a set of N linear equations in N unknowns where N is the number of nodes. This set of linear equations can be solved for the unknown parameters by Gaussian elimination or some more efficient scheme. In the LAKE model, the variables are the first derivatives of pollutant concentration with respect to time, C_t, at each of the nodal points. The set of equations to be solved can be represented as $AC_t = B$ where A is an N x N matrix, C_t is a vector containing the time derivatives of pollutant concentration at each of the N nodes, and B is a vector of N constants. After the results of the element-by-element integrations are assembled into the $AC_t = B$ equation set, each equation (row) represents integration of the governing equation over all elements centered around a single node. This simplifies the problem of providing for derivative boundary conditions. However, in this first generation lake model it is assumed that neither pollutant nor dissolved oxygen can move across the lake or stream boundaries. Therefore, the gradients for pollutant and dissolved oxygen must be zero at the lake boundaries. A second generation model might provide for a flux of pollutant across the lake boundaries.

Solution of the $AC_t = B$ set of equations yields values for C_t at each node at each point in time. A simple explicit time integration would be to multiply C_t, evaluated at the earlier time point, by the time increment, DT, and add the result to the previous value of C to find the new value for C. This crude scheme, however, can lead to serious computational error. The finite element method allows the use of a simple implicit integration scheme in the time domain in which C_t is evaluated at the mid-point of the time increment and multiplied by DT to find the incremental changes in C at each nodal point. After generation of the matrices involved is discussed, the time integration scheme will be presented.

If C is taken as the concentration (gm/m^3) of BOD, the first derivative of BOD with respect to time on the left of equation (15) is approximated as $C_{t1}u_1 + C_{t2}u_2 + C_{t3}u_3$ where 1, 2 and 3 are node numbers associated with a single element. Multiplying by the weighting function U and integrating over a single element give the following result.

$$h^* \int (u_1 + u_2 + u_3)\,(C_{t1}u_1 + C_{t2}u_2 + C_{t3}u_3)dA$$

Since the integral $\int u_i u_j dA$ equals A/12 when $i \neq j$ and A/6 when $i = j$, where A is the area of the element, relationships resulting from integration over each element are found as shown below. These are later assembled into the A matrix at the row and column numbers associated with the nodes for the element. The depth on the left of Equation (15) is taken as the average depth over the element equal to the mean of the depths at the bounding nodes. The average depth is designated as h* in the following equations and as HBAR in the computer program.

		VALUE OF THE EXPRESSION $h^*\int u_i u_j dA$		
		Column Numbers		
		1	2	3
Row Numbers	1	Ah*/6	Ah*/12	Ah*/12
	2	Ah*/12	Ah*/6	Ah*/12
	3	Ah*/12	Ah*/12	Ah*/6

The A matrix, called ARRAY (I,J) in the computer program, depends only on the discretization scheme laid out on the lake surface. At each time point, the A matrix is multipled by the C_t vector and set equal to the terms shown on the right of Equation (15), representing advection, dispersion, biodegradation and point sources. At any time point, all of the variables on the right of Equation (15) are constant, supplied either as input or from the results of the previous time step integration. For example, the depth (h) at each nodal point, the fluid velocity components (v & w), the dispersion coefficients, the biodegradation constant (K_r) and the point sources of pollutants are supplied as input and remain constant throughout the integration over time. The concentration of pollutant (C) at each node is supplied initially as input and at later times from the results of the time integration. The constants on the right of Equation 15 are most conveniently generated by development of several matrices which can then be multiplied by the C vector to produce the required constant values. For example, the biodegradation term can be represented as $-K_rAC$ where A is the ARRAY matrix, K_r is a constant supplied as input and C is the vector of nodal pollutant concentrations at each of the N nodes. Similarly, an F matrix can be found for advection and an E matrix for dispersion, both of which can be multiplied by the current C vector to evaluate the right side of equation (15) at any time point.

In order to develop the F matrix for advection consider the first two terms on the right of Equation 15. If the differentiation indicated is carried out, four of the resulting terms can be set equal to zero because the net flow of water into any column is known to be zero. For advection only, then, the simplified relationship is written as: $h^*C_t = -h(vC_x + wC_y)$. To carry out the integration over a single element make the following substitutions: $h = h_1u_1 + h_2u_2 + h_3u_3$; $V = v_1u_1 + v_2u_2 + v_3u_3$; $W = w_1u_1 + w_2u_2 + w_3u_3$; $C_x = C_1u_{x1} + C_2u_{x2} + C_3u_{x3}$ and $C = C_1u_{y1} + C_2u_{y2} + C_3u_{y3}$. The integral to be evaluated is written as: $-\int(hvC_x + hwC_y)(u_1 + u_2 + u_3)dA$. The result of the integration are shown in the following table where the column and row numbers correspond to the bounding node numbers for the element being considered.

	1	2	3
1	$A(u_{x1}(V\bullet G_1)+u_{y1}(W\bullet G_1))$	$A(u_{x1}(V\bullet G_2)+u_{y1}(W\bullet G_2))$	$A(u_{x1}(V\bullet G_3)+u_{y1}(W\bullet G_3))$
2	$A(u_{x2}(V\bullet G_1)+u_{y2}(W\bullet G_1))$	$A(u_{x2}(V\bullet G_2)+u_{y2}(W\bullet G_2))$	$A(u_{x2}(V\bullet G_3)+u_{y2}(W\bullet G_3))$
3	$A(u_{x3}(V\bullet G_1)+u_{y3}(W\bullet G_1))$	$A(u_{x3}(V\bullet G_2)+u_{y3}(W\bullet G_2))$	$A(u_{x3}(V\bullet G_3)+u_{y3}(W\bullet G_3))$

These 3x3 matrices are then assembled into the large NxN matrix called F. The row and column numbers shown as 1, 2 and 3 in the table shown above are actually the numbers of the bounding nodes for each element.

In the table shown above A is the area of the element over which the integration is being performed. V, W, G_1, G_2 and G_3 are all three-component vectors. For example, V has components v_1, v_2 and v_3 where the subscripts are the numbers of the bounding nodes. The same is true for the vector W. The three components of G_1 are $(h_1/10 + h_2/30 + h_3/30)$, $(h_1/30 + h_2/30 + h_3/60)$ and $(h_1/30 + h_2/60 + h_3/30)$. Similarly, for G_2 the three components are $(h_1/30 + h_2/30 + h_3/60)$, $(h_1/30 + h_2/10 + h_3/30)$ and $(h_1/60 + h_2/30 + h_3/30)$ and for G_3 the components are $(h_1/30 + h_2/60 + h_3/30)$, $(h_1/60 + h_2/30 + h_3/30)$ and $(h_1/30 + h_2/30 + h_3/10)$. V•G and V•W indicate the dot or scalar product of the two vectors. Since all of the terms of the F matrix depend only on the geometry of the lake and the velocity components at each node, the F matrix can be multiplied by the C vector at each time point to solve equation (15).

To develop the E matrix, which represents dispersive transport within the lake, the differentiation indicated in the third, fourth and fifth terms of Equation 15 must first be carried out. When this is done, second derivatives of C with respect to x and y appear and Green's Theorem must be applied to eliminate the second derivatives. The resulting integral equation to be evaluated over each element for dispersive transport is written as follows: $h^*C_t = -E_{xx}\int C_x u_x h\ dA - E_{yy}\int C_y u_y h\ dA - E_{xy}\int (C_x u_y + C_y u_x) h\ dA$. The resulting 3x3 matrices are again assembled into the larger NxN matrix which is called E. The general term of the 3x3 matrix can be written as follows, where i is the row number and j is the column number: $h^*A(E_{xx}u_{xi}u_{xj} + E_{yy}u_{yi}u_{yj} + E_{xy}(u_{xi}u_{yj} + u_{yi}u_{xj}))$. The E matrix is generated only once since it depends only on the geometry of the lake and the dispersion coefficients. The constant terms (3, 4 and 5) on the right of Equation 15 are then found by multiplying the E matrix by the C vector available from the last integration step or from input.

Integrating the last term of Equation 15 over an element clearly results in the rate of pollutant discharge (P) over the time interval being considered. So, the contribution of the last term to the constant on the right of Equation 15 is simply the input value of P.

Finally, after having generated the matrices A, F and E, and the vector P, the governing equations (15 and 16) can now be written in the form of matrix equations as follows.

$$AC_t = -FC - EC - K_r AC + P \qquad (17)$$

$$AD_t = -FD - ED - K_L AD + K_d AC \qquad (18)$$

In these equations A, F and E are NxN matrices where N is the number of nodes. C, D, C_t and P are all vectors with N components. At each time point values for all variables on the right of Equations 17 and 18 will be known and the matrix multiplication can be carried out yielding N equations in N unknowns.

With the governing equations expressed in matrix form as shown in Equations 17 and 18, an implicit method of integration in the time domain becomes possible. For example, if C_o is taken as the concentration of pollutant at the earlier time point and C_1 is the corresponding value at the later time point, Equation 17 can be written as follows:

$$A(C_1-C_o)/DT = -F(C_1 + C_o)/2 - E(C_1 + C_o)/2 - K_rA(C_1 + C_o)/2 + P$$

By rearranging this relationship, a matrix equation of the form $GC_1 = BC_o$ can be found which can then be solved directly for C_1. For Equation 17, the values for the constants G and B are: $G = A(1/DT + K_r/2) + F/2 + E/2$; $B = A(1/DT - K_r/2) - F/2 - E/2$. Equation 18 can be similarly rearranged to yield a matrix equation of the form $GD_1 = K_dA(C_1 + C_o)/2 + BD_o$. Values for G and B in this equation are the same as those found using Equation 17 except that K_L must be substituted for K_r. Notice that if the integration for C is performed first, the value for C_1 will be available to use in the solution for dissolved oxygen deficit (D).

If the nodal numbers around each element are selected carefully, the coefficients of the N equations to be solved simultaneously will form a banded matrix. Logic is contained in the program LAKE to form the N equations into a form suitable for solution as a balanced matrix. This involves transforming the diagonal into a center column in the transformed matrix. The subroutine for solving this transformed matrix is similar to the subroutine SLBS given in Brebbia (1978). The computer program for LAKE is enclosed in the Appendix.