Response of a Pile to Impinging Seismic Waves using a Poroelastic Boundary Element Method

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Abstract

A frequency domain boundary element method is presented to study the seismic response of single piles to vertically propagating shear waves within a poroelastic medium. Nondimensional kinematic displacement factors are presented for the elastic and poroelastic responses of the pile. The treatment of the soil as a multi-phase poroelastic material is found to result in reduced displacements due to additional damping from the fluid phase.

Introduction

The dynamic response of piles and drilled shafts subjected to impinging seismic waves is of critical importance in the study of soil-structure interaction. Traditionally, such problems have been studied using linear elastic analysis. There has been little emphasis on modeling the soil as a multi-phase porous material. The present paper will model the soil as a saturated, poroelastic material governed by Biot's poroelastic theoretical models. The Boundary Element Method (BEM) will be utilized for the solution scheme since it has established itself as the ideal method for the study of wave propagation in infinite and semi-infinite media such as soils and rocks. The seismic behavior of a single pile foundation subjected to vertically incident shear waves is investigated in this paper.

Governing Equations and Integral Formulation

Dynamic poroelastic analysis finds its roots in the general theory of Biot (1956) governing the behavior of two-phase fluid-filled media such as soils. The inherent complexity of the associated phenomena prevents the development of analytical solutions for all but the simplest of geometries and boundary conditions. Consequently, the use of numerical methods such as finite element and boundary element methods becomes imperative. In particular, the boundary element method (BEM) is quite attractive for semi-infinite media, at least in situations where the media can be modeled as piecewise homogeneous. Early work on the fundamental solutions and BEM formulations for dynamic poroelasticity may be attributed to Bonnet (1987), Boutin et al. (1987), Predeleanu (1984), Cheng et al. (1991) and Dominguez (1991). Recently, Chen and Dargush (1995) presented a Laplace domain transient BEM for both two- and three-dimensional dynamic poroelastic analysis. Dargush and Chopra (1996) presented a frequency domain solution for the dynamic response of circular foundations using a poroelastic formulation and Dargush and Chopra (1998) applied the Laplace domain formulation to the seismic response of sedimentary basins. The present work is based on these previous developments and is extended to the study of the response of single piles to impinging seismic waves.

The governing equations of the theory of dynamic poroelasticity may be reduced in the transform domain to the following equations of momentum and mass balance:

$$\mu \tilde{u}_{i,jj} + (\lambda + \mu)\tilde{u}_{j,ij} + \omega^2 \rho_1 \tilde{u}_i - \alpha_1 \tilde{p}_{,i} + \tilde{f}_i = 0 \tag{1a}$$

$$\zeta \tilde{p}_{,ii} - (i\omega / Q)\tilde{p} - i\omega\alpha_1 \tilde{u}_{i,i} + \tilde{\psi} = 0 \tag{1b}$$

in which all Latin indices assume the values 1,2,3 for three-dimensional domains and a superposed tilde is used to denote a Fourier transformed variable. Commas indicate derivatives with respect to spatial coordinates. In equation (1), u_i is the displacement of the soil skeleton, p is the excess pore water pressure, λ and μ are the drained Lame constants and k is the coefficient of permeability. The symbols ψ and f_i denote the volumetric body source rate and body force terms. Based on Biot (1956), $\alpha_1 = \alpha - i\omega\rho_f\zeta$, $\rho_1 = \rho - i\omega\rho^2{}_f\zeta$ and $\zeta = (1/k + i\omega\rho_f / n)^{-1}$ with the porosity of the soil as n. The quantities ρ and ρ_f are total and fluid densities, respectively. Remaining Biot parameters α and Q are compressibility parameters and can be related to Skempton's pore pressure parameter B and the undrained Poisson's ratio ν_u.

An exact integral representation may be developed from equation (1) in a systematic manner as shown, for example, in Chen and Dargush (1995). In the absence of distributed body forces or body sources, the resulting boundary integral equation for a poroelastic domain V with a bounding surface S, may be expressed as:

$$C_{\beta\alpha}(\xi)\tilde{u}_\beta(\xi;\omega) = \int_S \left\{ G^*{}_{\beta\alpha}(x,\xi;\omega)\tilde{t}_\beta(x;\omega) - F^*{}_{\beta\alpha}(x,\xi;\omega)\tilde{u}_\beta(x;\omega) \right\} dS(x) + \sum_{n=1}^{N} G^*{}_{i\alpha,j}(x_n,\xi;\omega)\tilde{M}_{ij}(x_n;\omega) \tag{2}$$

where $t_\beta = \{\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{q}\}^T$ and $u_\beta = \{\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{p}\}^T$ are the generalized tractions and the generalized displacements, respectively and $\tilde{q}$ denotes the normal volumetric flux. All Greek indices assume the values 1,2,3,4 for three-dimensional analysis. The terms involving M_{ij} represent the contributions of the seismic source (Aki and Richards, 1980), which is modeled as the summation of discrete moment tensors located at the N points x_n. The displacement and traction kernels are denoted by G^*_{ij} and F^*_{ij} respectively. Matrix components $C_{\beta\alpha}$ depend upon the local geometry at the point ξ and reduce to $\delta_{\beta\alpha}/2$ at a smooth boundary where $\delta_{\beta\alpha}$ represents a generalized Kronecker delta function.

The dynamic poroelastic kernels in three-dimension are provided in Chen and Dargush (1995). A closer examination of the displacement kernels reveals a presence of a third wave in addition to the two body waves. The third wave corresponds to another compressional wave adding to the pressure and shear wave from elastic wave propagation theories. It is notable in equation (2) that for homogeneous poroelastic regions, the entire integral equation is surface-only in nature and does not involve variables at any interior points in the solution process. The only volume contributions come from treating the seismic moment sources as body source terms, which are assumed to be completely known apriori. Thus, the introduction of the poroelastic fundamental solutions have reduced the dimensionality of the problem by one. Piecewise homogeneous regions may be accommodated using multiple regions, which is the technique used to model the pile embedded within the soil mass.

Numerical Applications

Problem Description – Single Pile

The Laplace domain poroelastic BEM formulation is now used to study the response of a single pile to a vertically propagating harmonic shear wave. The impinging wave produces a horizontal oscillation of $U_{ff}\ exp\ (i\omega t)$ at a “free-field” point on the surface of the soil at a location unaffected by the presence of the pile. The following non-dimensional geometric property is selected for the analysis: L/d = 40, where L is the length of the pile and d is its diameter. The poroelastic material is assumed to have the following properties: $\nu_S = 0.4$, $\mu_S = 0.1786$, porosity $n_S = 0.3$, $\alpha = 1.0$, $Q = 1.31$, $\rho = 1/2$ and $k_S = 1.1\text{x}\ 10^{+1}$. The properties of the pile are: $\nu_P = 0.4$, porosity $n_P = 0.3$, $\alpha = 1.0$, $Q = 3.0$, $\rho = 1/2$ and $k_P = 1.1\text{x}\ 10^{-3}$. Other related properties are non-dimensional in nature such as: $\rho_S/\rho_P = 0.7$ and E_P/E_S = 1,000 and 10,000. In addition, a hysteretic damping factor β of 0.05 is included for the soil.

A sensitivity analysis is first conducted to ensure that the seismic source is placed deep enough to produce almost uniform displacements at the soil surface. The mesh is terminated far enough from the pile location to have a minimal effect on its response.

Results

The seismic response of the single pile to the S wave is portrayed using kinematic displacement factors, which is defined as the magnitude of pile head displacement (either elastic or poroelastic) normalized with respect to the elastic free-field displacement at the soil surface. Initially, an elastic BEM analysis was conducted to compare the results with published literature to gain confidence in the seismic source technique using moment tensors. The BEM results from an elastic analysis for E_P/E_S = 1,000 and 10,000 are shown in Figure 1 along with the results from Fan et al. (1990). The correlation is fairly good considering the seismic source is modeled using one source point at a depth of $D_{source}/d = 200$, where D_{source} is depth of the source below soil surface.

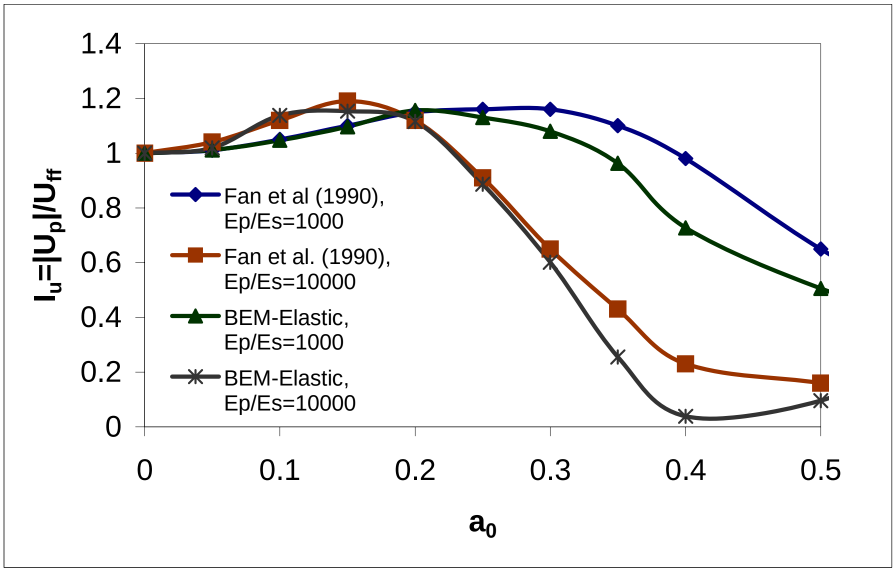

Figure 1. Response of a single pile to impinging seismic SV wave – elastic BEM solution compared with Fan et al. (1990).

Next, the poroelastic BEM formulation is used to study the seismic response of the same pile where the soil and pile are modeled as poroelastic media. The poroelastic response is significantly damped due to the presence of the fluid phase and this is evident from the results of the present analysis. Figure 2 shows the normalized kinematic displacement factor for poroelastic behavior in comparison with the corresponding elastic response for the modulus of elasticity ratio of 1000 as a function of the soil permeability *k*. Lastly, the same response is presented in Figure 3 for $E_P/E_S = 10000$. In each case, the amplitude and duration of the poroelastic response is significantly lower.

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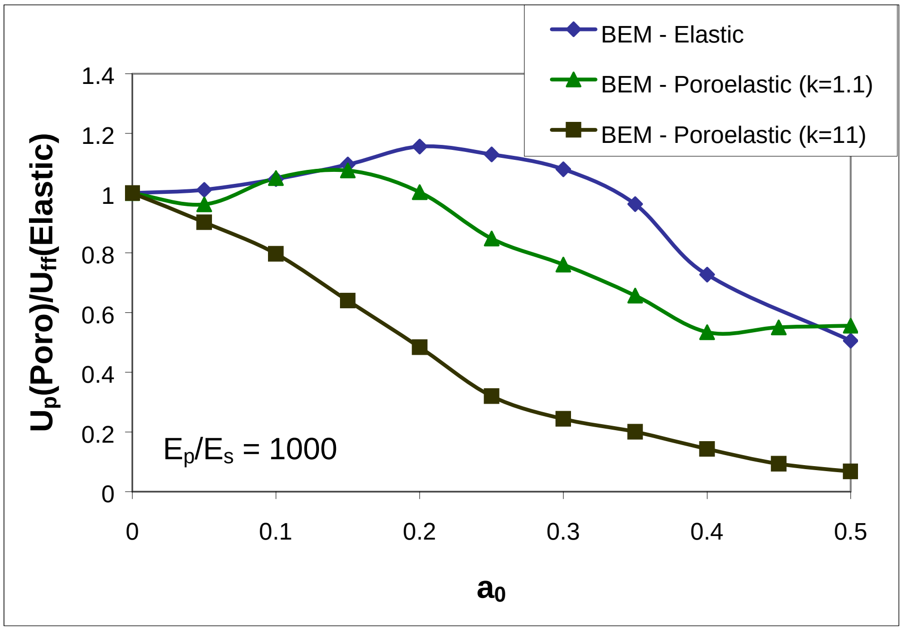

Figure 2. Poroelastic Response of a Single Pile subjected to an impinging SV wave for Ep/Es = 1000

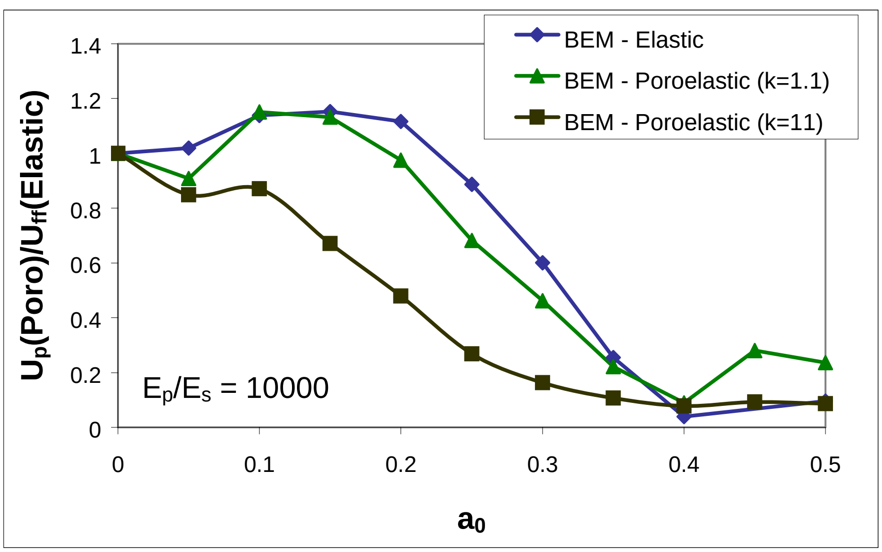

Figure 3. Poroelastic Response of a Single Pile subjected to an impinging SV wave for $E_p/E_s = 10000$.