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Dynamic response of embedded circular foundations using a poroelastic BEM

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ABSTRACT: The dynamic response of embedded circular shallow foundations subjected to time-harmonic vertical excitations is studied in this paper. A dynamic poroelastic boundary element method (BEM) is used for the analysis of such foundations. An axisymmetric BEM formulation is developed which offers reduction in the dimensionality of the problem. The paper aims to extend the previous work by the authors on the dynamic response of circular footings on the surface of poroelastic soil media to account for foundation embedment. The effect of different embedment ratios and permeability values of the soil is investigated.

1 INTRODUCTION

The dynamic behavior of poroelastic media, such as soils and rocks, is often determined on the basis of a single-phase linear elastic model in the study of soil-structure interaction. The boundary element method (BEM) has been applied with much success to the solution of elastodynamic soil-structure interaction. The present paper aims at modeling the soil in a more realistic manner by accounting for the inertia effects within the pore water phase and the interaction of the soil skeleton and the pore water under dynamic loads. Although the response within an infinite medium may often be described by a single phase model with suitable damping, the effect of layering of the soil adds more complexity and requires a more detailed study. BEM is ideally suited for the study of the dynamic behavior of semi-infinite media. The axisymmetric dynamic poroelastic BEM developed by the authors in a previous paper (Dargush and Chopra, 1996) for the response of surface footings is applied in this work to account for the effects of foundation embedment.

Dynamic poroelastic analysis has its basis in the effective stress theory of Terzaghi. Biot (1956) extended the work of Terzaghi to a general theory governing the behavior of two-phase fluid-filled materials such as soils. The correctness of this theory in the linear range has been confirmed by other approaches such as a two-scaled analysis of

Navier-Stokes equations and the theory of mixtures. The coupled phenomenon, however, precludes the development of analytical solutions for all but the simplest of geometry and boundary conditions. The finite element method has also been used with Biot's theory for poroelastic applications.

The initial application of BEM to dynamic poroelastic analysis involved the use of six unknowns (solid skeleton displacements and the average relative solid-fluid displacement). Cheng et al. (1991) and Dominguez (1992) then developed two-dimensional frequency domain BEM solutions for the dynamic case using only four independent variables (the solid displacements and pore pressure) in the governing equations. Subsequently, Chen and Dargush (1994) presented a complete transient BEM for both 2-D and 3-D dynamic poroelastic analysis. Recently, Dargush and Chopra (1996) have presented a BEM for axisymmetric dynamic problems and have studied the response of circular footings on the surface of a poroelastic medium. The present work applies this formulation to the analysis of the response of embedded circular foundations under vertical excitation. The effect of different embedment ratios and various permeability values for the soil is investigated. An attempt is made to study the dynamic behavior of embedded foundations in a semi-infinite half-space and a layer of soil overlying a hard stratum.

2 BOUNDARY ELEMENT FORMULATION

The governing equations for dynamic poroelastic analysis based upon Biot's theory may be expressed in frequency domain as balances of momentum and mass of a fluid-filled medium, as follows:

$$\mu \tilde{u}_{i,jj} + (\lambda + \mu) \tilde{u}_{j,i,j} + \omega^2 \rho_1 \tilde{u}_i - \alpha_1 \tilde{p}_{,i} + \tilde{f}_i = 0 \quad (1a)$$

$$\zeta \tilde{p}_{,ii} - (i\omega / Q) \tilde{p} - i\omega \alpha_1 \tilde{u}_{i,i} + \tilde{\psi} = 0 \quad (1b)$$

where u_i is the displacement of the solid skeleton, p denotes the pore water pressure. The parameters λ and μ are the drained Lamé constants, κ is the permeability coefficient of the soil (i.e. $\kappa = k / \eta$) where η is the fluid viscosity and k the specific permeability of the soil. Other Biot parameters include:

$$\begin{aligned} \alpha_1 &= \alpha - i\omega \rho_f \zeta; \quad \rho_1 = \rho - i\omega \rho_f^2 \zeta; \\ \zeta &= (1 / \kappa + i\omega m)^{-1} \end{aligned} \quad (2)$$

where ω is the rotational frequency. The effective permeability becomes a complex valued function of ω . The quantities ρ and ρ_f denote the total and fluid densities respectively. The parameters α and Q are parameters accounting for the material compressibilities (Dargush and Chopra, 1996) and ψ and f_i are the volumetric body source rate and body force respectively. Lastly, m is a parameter arising from the generalized Darcy's Law and is related to the inertial effects of the fluid behavior. The superposed tilde denotes variables transformed to the frequency domain.

An exact boundary integral equation, in the absence of body forces and sources, may be expressed as follows for a poroelastic volume V bounded by a surface S (Chen and Dargush, 1995):

$$\begin{aligned} C_{\beta\alpha}(x) \tilde{u}_\beta(x;w) &= \int_S [G_{\beta\alpha}(x,x;w) \tilde{t}_\beta(x;w) \\ &\quad - F_{\beta\alpha}(x,x;w) \tilde{u}_\beta(x;w)] dS(x) \end{aligned} \quad (3)$$

where

$$\tilde{t}_\beta = \{\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{q}\}^T \text{ and } \tilde{u}_\beta = \{\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{p}\}^T$$

are the generalized tractions and the generalized displacements. $G_{\beta\alpha}$ is the displacement kernel and the traction kernel $F_{\beta\alpha}$ is derived from the displacement kernel by using the stress-strain and the strain-displacement relations. The matrix $C_{\beta\alpha}$ depends upon the local geometry of the boundary. Detailed expressions for the kernels are provided in Chen and Dargush (1994) for the 3-D case.

The axisymmetric kernels are presented in Dargush and Chopra (1996) and are obtained by the integration of the 3-D kernels in the circumferential direction from 0 to 2π and suitably transforming all variables to a cylindrical coordinate system. These kernels exhibit singular behavior as discussed in Chen and Dargush (1994). The remaining portion of the dynamic poroelastic kernels are transient and non-singular. The generalized axisymmetric case involving axisymmetric geometry under non-axisymmetric loading and boundary conditions is handled by decomposing the displacement and traction fields into symmetric and antisymmetric components using Fourier Series expansion. This leads to two decoupled integral equations for each harmonic mode, as detailed in Dargush and Chopra (1996).

Although Equation (3) is an exact boundary integral equation, it is difficult to solve analytically for anything but very simple problems. Hence, the next step is to discretize this equation numerically. Both temporal and spatial discretization is carried out in the standard manner. The kernels functions become explicitly defined complex quantities and the generalized displacements and tractions represent complex amplitudes. For the time-harmonic excitation case, the boundary integral equations (3) are solved in the frequency domain. Spatial collocation and numerical integration of the non-singular and singular kernels, leads to the discretized form of (3). These equations are written for each boundary node and arranged into a system of algebraic equations in a standard BEM manner. The boundary conditions are then applied which leads to a unsymmetric complex system matrix $[A]$ such that:

$$[A]\{X\} = \{b\} \quad (4)$$

where $\{b\}$ is the vector of known quantities. This equation may be solved for the unknown variables $\{X\}$ on the boundary. Details of the numerical

implementation may be found in previous works on axisymmetric BEM. It should be noted that no special treatment is required for the incompressible response under undrained conditions nor for the satisfaction of radiation boundary conditions.

3 NUMERICAL APPLICATIONS

Dynamic Response of a Circular Footing Embedded in a Poroelastic Soil Layer

The BEM formulation is ideally suited for the study of the dynamic response of foundations. As an illustration, the method is now applied to investigate the vertical compliance of a smooth, rigid, impermeable circular footing of radius R embedded within a poroelastic layer of soil overlying a hard stratum. The layer is modeled as a two-phase poroelastic medium. The soil properties are selected from Dargush and Chopra (1996) as: Poisson's Ratio $\nu = 1/3$; damping coefficient $D = 0.05$; Porosity $n = 0.30$; the shear wave velocity $c_s = 152$ m/s; the pore water wave velocity $c_w = 1439$ m/s; solid grain weight density 25.9 kN/m^3 ; pore water unit weight 9.81 kN/m^3 .

Since actual soils have a finite permeability, thus for a given value of κ , a second time scale enters the picture and the response, in general, depends upon a characteristic length (e.g., R). Different degrees of permeability are considered and a dimensionless parameter χ is introduced where $\chi = c_v / (c_s R)$ and c_v is the coefficient of consolidation from the quasistatic formulation which is related to the permeability as described in Dargush and Chopra (1996). The thickness of the single homogeneous elastic layer of soil is expressed as H and $H/R = 2$ for the current study. The role of layer resonance in the overall footing response is more readily seen in plots involving the dynamic compliance of the foundations. The compliance values are normalized using the real static stiffness for the embedded foundations under vertical excitation. Four levels of embedment, corresponding to $d/R = 0, 0.5, 1.0$ and 1.5 , are considered where d is the depth of embedment. In performing the normalization, the following stiffness values were determined from fully drained BEM analyses: $K_o = 10.2 \mu R$ for $d/R = 0$; $K_o = 14.3 \mu R$ for $d/R = 0.5$; $K_o = 19.3 \mu R$ for $d/R = 1.0$ and $K_o = 32.8 \mu R$ for $d/R = 1.5$. Figure 1 shows the variation of the normalized compliance

with the dimensionless frequency a_o where $a_o = \omega R / c_s$. The frequency dependence and the dynamic amplification of the foundation response is clearly evident from the figure. With increasing depths of embedment, the dynamic amplification begin to shift to the right to higher frequencies. This phenomenon is associated with the propagation of generalized Rayleigh waves within the elastic soil layer.

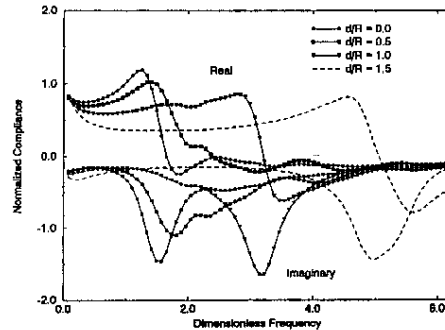


Figure 1: Effect of Embedment on the Response of a Circular Footing Embedded in a Layer ($\chi = 0.067$)

Next, the effect of various permeability values of the layer are considered. Figure 2 presents the normalized compliance of the foundation for varying levels of the non-dimensional parameter χ for a surface footing (i.e. $d/R = 0$). Large values of χ are associated with very permeable soils or very small footings resting on thin soil layers. For example, $\chi = 0.67$ corresponds to the soil with a permeability coefficient of 0.006 m/s for a footing of 1 m radius. For this case, the frequency range covered in Fig. 2 is from zero to 152 Hz. The solutions obtained from completely drained and undrained elastic analyses are also shown in the figure. The result for $\chi = 0.0067$ coincided with the undrained response and is not shown for clarity. For a given footing size, the compliance for a higher permeability value follows the drained response in the low frequency domain, while impermeable cases tend toward undrained behavior throughout the frequency range considered. The dynamic amplification of the response due to surface wave propagation near the frequency $a_o \approx \pi/2$ is once again evident.

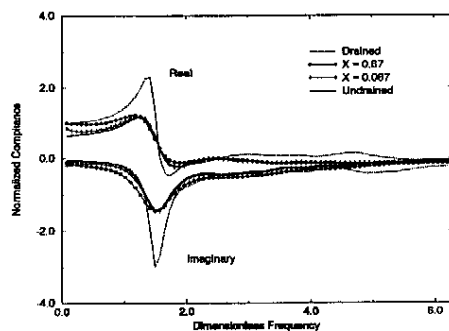


Figure 2: Effect of Permeability on the Response of a Circular Footing on the Surface ($d/R = 0$)

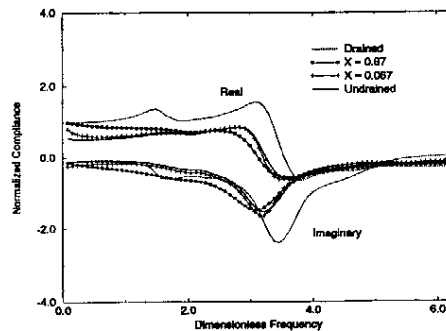


Figure 3: Effect of Permeability on the Response of an Embedded Circular Footing ($d/R = 1.0$)

Figure 3 presents the effect of embedment on the corresponding variation of normalized compliance at different permeability levels. The intermediate case of $d/R = 1.0$ is plotted as an illustration. It is notable that the amplitude of the dynamic amplification is somewhat reduced due to the presence of adjoining soil mass for an embedded footing. In addition, the resonant frequency of the layer is shifted to $\alpha_0 \approx \pi$ for this case. Once again, at low frequencies, the response for a higher χ is closer to the drained case but tends to shift towards the undrained case at higher frequencies. The response for lower χ value tends towards the undrained response at all α_0 levels.

Dynamic Response of a Circular Footing Embedded in a Poroelastic Half Space

The vertical response of the same circular footing embedded in a semi-infinite homogeneous poroelastic half space is now studied. Figure 4 presents the normalized compliance of the footing as a function of different embedment ratios. However, the dynamic behavior for this case is significantly different from the layered soil. There is very little effect of embedment on the response while no evidence of dynamic amplification is evident. As expected, at higher frequency the dynamic compliance of the footing embedded in a half-space tends towards zero in a monotonic fashion. Comparisons with the elastodynamic case for the embedded foundations will be presented in a forthcoming work.

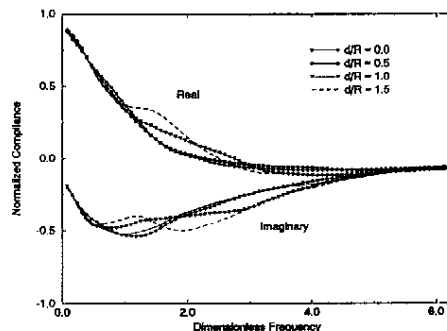


Figure 4: Dynamic Response of a Circular Footing Embedded in a Poroelastic Half-Space ($\chi = 0.067$)

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